Real Numbers 1

NCERT SOLUTIONS

What's inside

– In-Chapter Q's (solved)

– Textbook Exercise Q's (solved)

Exercise – 1.1

1. Express each number as a product of its prime factors :

(i) 140 **(ii) 156** **(iii) 3825**

(iv) 5005 **(v) 7429**

Sol. : (i) 140

2	140
2	70
5	35
7	7
	1

Hence, prime factors of 140 $= 2 \times 2 \times 5 \times 7$

$= 2^2 \times 5 \times 7$

(ii) 156

2	156
2	78
3	39
13	13
	1

Hence, prime factors of 156 $= 2 \times 2 \times 3 \times 13$

$= 2^2 \times 3 \times 13$

(iii) 3825

3	3825
3	1275
5	425
5	85
17	17
	1

Hence, prime factors of 3825

$= 3 \times 3 \times 5 \times 5 \times 17$

$= 3^2 \times 5^2 \times 17$

(iv) 5005

5	5005
7	1001
11	143
13	13
	1

Hence, prime factors of 5005

$= 5 \times 7 \times 11 \times 13$

(v) 7429

17	7429
19	437
23	23
	1

Hence, prime factors of 7429

$= 17 \times 19 \times 23$

2. Find the L.C.M. and H.C.F. of the following pairs of integers and verify that L.C.M. × H.C.F. = product of the two numbers .

(i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

Sol. : (i) 26 and 91

Prime factors of 26 $= 2 \times 13$

Prime factors of 91 $= 7 \times 13$

H.C.F. = the product of the smallest power of common factors of the numbers

Hence, H.C.F. = 13

L.C.M. = the product of the greatest power of each prime factors associated with the numbers.

L.C.M. $= 2 \times 7 \times 13 = 182$

Now,

H.C.F. × L.C.M. $= 13 \times 182 = 2366$

andproduct of both the numbers $= 26 \times 91$

$= 2366$

Hence, H.C.F. × L.C.M. = Product of both the numbers.

(ii) 510 and 92

Prime factors of 510 $= 2 \times 3 \times 5 \times 17$

Prime factor of 92 $= 2 \times 2 \times 23$

$= 2^2 \times 23$

∴ H.C.F. = the product of the smallest power of common factors of the numbers

Hence, H.C.F. = 2

L.C.M. = the product of the greatest power of each of the prime factors associated with the numbers.

Hence, L.C.M. $= 2^2 \times 3 \times 5 \times 17 \times 23$

$= 23460$

Now,

H.C.F. × L.C.M. $= 2 \times 23460$

$= 46920$

andProduct of two numbers $= 510 \times 92$

$= 46920$

∴ HCF × LCM = Product of two numbers.

(iii) 336 and 54

Prime factors of 366 $= 2^4 \times 3 \times 7$

Prime factors of 54 $= 2 \times 3^3$

H.C.F. = the product of the smallest power of common factors of the numbers

$= 2 \times 3 = 6$

L.C.M. = The product of the greatest power of each of the prime factors associated with the numbers.

$= 2^4 \times 3^3 \times 7$

$= 3024$

Now, H.C.F. × L.C.M. = 6 × 3024 = 18144

and Product of two numbers = 336 × 54

= 18144

∴ H.C.F. × L.C.M. = Product of two numbers.

3. Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21 **(ii) 17, 23 and 29**

(iii) 8, 9 and 25

Sol. : **(i)** 12, 15 and 21

$$\begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

Hence, $12 = 2^2 \times 3$

$15 = 3 \times 5$

$21 = 3 \times 7$

∴ H.C.F. = 3

and L.C.M. $= 2^2 \times 3 \times 5 \times 7$

$= 420$

(ii) 17, 23 and 29

∵ 17, 23 and 29 are prime numbers.

∴ H.C.F. = 1

and L.C.M. = 17 × 23 × 29

= 11,339

(iii) 8, 9 and 25

$$\begin{array}{r|l} 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

Hence, $8 = 2^3$

$9 = 3^2$

$25 = 5^2$

∴ H.C.F. = 1

and L.C.M. $= 2^2 \times 3^2 \times 5^2 = 1800$

4. Given that H.C.F. (306, 657) = 9, find L.C.M. (306, 657).

Sol. : Given,

H.C.F. (306, 657) = 9

Given numbers = 306, 657

$\therefore$ H.C.F. (306, 657) × L.C.M. (306, 657)

= Product of two numbers

9 × L.C.M. (306, 657) = 306 × 657

L.C.M. (306, 657) = $\frac{306 \times 657}{9}$

= 306 × 73

L.C.M. (306, 657) = 22,338

5. Check whether 6^n can end with the digit 0 for any natural number n.

Sol. : If unit digit of any number is 0 then it is always divisible by 5.

Let unit digit of 6^n is 0 then it should be divided by 5, it is only possible when one of the prime factors of 6^n is 5.

Now, $6 = 2 \times 3$

$6^n = (2 \times 3)^n$

$= 2^n \times 3^n$

Thus, 5 is not present in the factors of 6^n. Hence, there is no value for $n \in$ N for which unit digits is 0.

6. Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.

Sol. : (i) Given, 7 × 11 × 13 + 13

= 13 × (7 × 11 + 1)

= 13 × (77 + 1)

= 13 × 78

= 13 × 13 × 3 × 2

It means factors of given expression are 2, 3 and 13 respectively. Hence, it is composite number.

(ii) 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5

= 5(7 × 6 × 4 × 3 × 2 + 1)

= 5 × (1008 + 1)

= 5 × 1009

Factors of given expression is 5 and 1009. Hence, it is composite number.

7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point ?

Sol. : Sonia takes time to drive one round = 18 min

Ravi takes time to drive one round = 12 min

If they both start at the same point and at the same time and go in the same direction. To find out the again meeting time at the starting point we need to find out the L.C.M. of 18 and 12.

So, $18 = 2 \times 3^2$

$$12 = 2^2 \times 3$$

L.C.M. = Product of the greatest power of each prime factors

$$= 2^2 \times 3^2 = 9 \times 4 = 36$$

Thus, they will meet after 36 min.

Exercise – 1.2

1. Prove that $\sqrt{5}$ is an irrational number.

Sol. : Let $\sqrt{5}$ is a rational number. It can be written as form $\frac{a}{b}$, where a and b are co-prime number.

Hence, $\sqrt{5} = \frac{a}{b}$

$$(\sqrt{5})^2 = \frac{a^2}{b^2}$$

$$5 = \frac{a^2}{b^2}$$

$$5b^2 = a^2$$

a^2 is divided by 5 then a is divided by 5.

Then we can write $a = 5m$, where m is any integer

Put $a = 5m$ in equation (i),

$$5b^2 = (5m)^2$$

$$\Rightarrow \quad 5b^2 = 25m^2$$

$$b^2 = 5m^2$$

b^2 is divided by 5, then b is also divided by 5. So, a and b have atleast one common factor but this contradicts the fact that a and b are co-prime numbers. We got this contradiction because we have mistakenly assumed that $\sqrt{5}$ is a rationl number.

Hence, it concludes that $\sqrt{5}$ is an irrational number.

2. Prove that $3+2\sqrt{5}$ is an irrational.

Sol. : Let $3+2\sqrt{5}$ is a rational number. It can be written as form $\frac{a}{b}$ where a and b are co-prime numbers.

So, $3+2\sqrt{5} = \frac{a}{b}$

$$2\sqrt{5} = \frac{a-3b}{b} \Rightarrow \sqrt{5} = \frac{a-3b}{2b}$$

$\because$ a and b are integers.

$\therefore$ $\frac{a-3b}{2b}$ is a rational number.

It means $\sqrt{5}$ is also rational number but we know that $\sqrt{5}$ is an irrational number. Hence, our assumption $3+2\sqrt{5}$ is a rational number is wrong.

Hence, $3+2\sqrt{5}$ is an irrational number.

3. Prove that the following numbers are irrational :

(i) $7\sqrt{5}$

(ii) $6+\sqrt{2}$

(iii) $\frac{1}{\sqrt{2}}$.

Sol. : (i) Let $7\sqrt{5}$ is a rational number which is in the form of $\frac{a}{b}$.

Where, $b \neq 0$

$$7\sqrt{5} = \frac{a}{b}$$

Squaring on both sides,

$$245 = \frac{a^2}{b^2}$$

$$a^2 = 245b^2 \qquad \text{...(i)}$$

a^2 is divisible by 245

then a will be divisible by 245

Let, $a = 245\ m$

By equation (i),

$$(245\ m)^2 = 245b^2 \qquad \text{...(ii)}$$

$\therefore$ b^2 is divisible by 245

then b will be divided by 245

by equation (i) and (ii), the common factor of a and b is 245. Which contradicts our assumption that a and b do not have any common factor other than 1. Hence, $7\sqrt{5}$ is irrational number.

(iii) Let $6+\sqrt{2}$ is a rational number.

So, it can be written as form $\frac{a}{b}$

$6+\sqrt{2} = \frac{a}{b}$, where a, b are co-prime numbers and $b \neq 0$.

$$\sqrt{2} = \frac{a}{b} - 6$$

$$\sqrt{2} = \frac{a-6b}{b}$$

$\therefore$ a, b, and 6 are integers, so $\frac{a-6b}{b}$ is a rational number. But by the contraction method. $\sqrt{2}$ is an irrational number. Hence, $6+\sqrt{2}$ is an irrational number.

(vi) Let $\frac{1}{\sqrt{2}}$ is a rational it can be written as form $\frac{a}{b}$.

$$\frac{1}{\sqrt{2}} = \frac{a}{b}$$

$$\frac{b}{a} = \sqrt{2}$$

$\frac{b}{a}$ is a rational number but this contradicts the fact that $\sqrt{2}$ is an irrational number.

Thus, $\frac{1}{\sqrt{2}}$ is an irrational number.

Polynomials 2

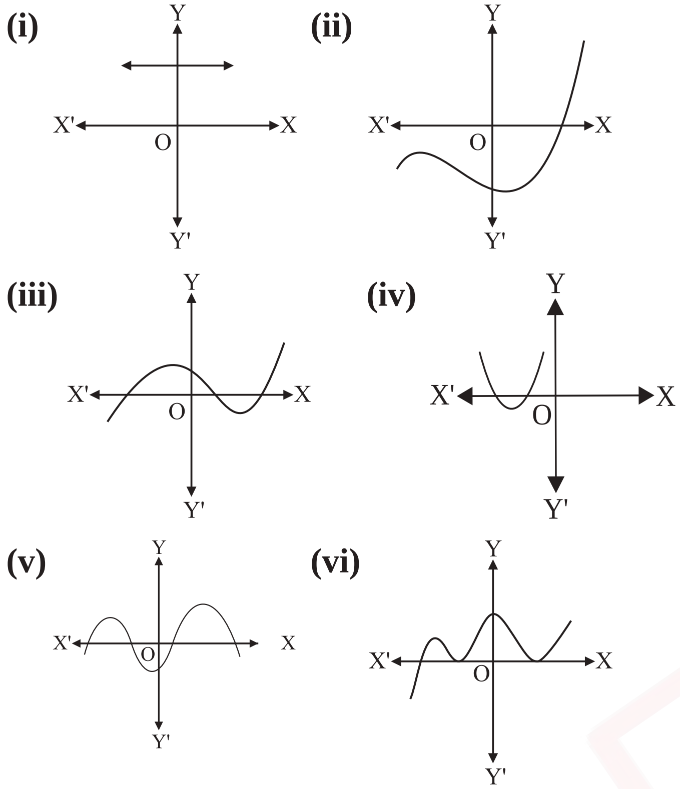
NCERT SOLUTIONS

What's inside

– *In-Chapter Q's (solved)*
– *Textbook Exercise Q's (solved)*

Exercise – 2.1

1. The graphs of $y = p(x)$ are given in fig. below, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.



Sol. : **(i)** None, because graph is not intersecting the at X-axis at any point.

(ii) 1, because graph intersects X-axis at only one point.

(iii) 3, because graph intersects X-axis at three points.

(iv) 2, because graph intersects X-axis at two points.

(v) 4, because graph intersects X-axis at four points.

(vi) 3 zeroes, because graph intersects X-axis at three points.

Exercise – 2.2

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients :

(i) $x^2 - 2x - 8$ **(ii) $4s^2 - 4s + 1$**

(iii) $6x^2 - 3 - 7x$ **(iv) $4u^2 + 8u$**

(v) $t^2 - 15$ **(vi) $3x^2 - x - 4$**

Sol. : **(i)** Let $p(x) = x^2 - 2x - 8$

$= x^2 - 4x + 2x - 8$

$= x(x - 4) + 2(x - 4)$

$= (x - 4)(x + 2)$

For finding the zeroes

Put, $p(x) = 0$

$(x - 4)(x + 2) = 0$

$x = 4, -2$

Verification :

Now, sum of zeroes $= 4 + (-2)$

$$= \frac{2}{1} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

Product of zeroes $= 4 \times -2 = \frac{-8}{1}$

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Hence, relationship between the zeroes and the coefficients is true.

(ii) Let $p(s) = 4s^2 - 4s + 1$

$= 4s^2 - 2s - 2s + 1$

$= 2s(2s - 1) - 1(2s - 1)$

$= (2s - 1)(2s - 1)$

$= (2s - 1)^2$

For finding the zeroes, put $p(s) = 0$.

$2s - 1 = 0$

$s = \frac{1}{2}$

Hence, zeroes of $p(s)$ are $\frac{1}{2}$ and $\frac{1}{2}$.

Verification :

Now, sum of zeroes

$$= \frac{1}{2} + \frac{1}{2} = 1 = \frac{4}{4}$$

$$= \frac{-(\text{Coefficient of } s)}{\text{Coefficient of } s^2}$$

Product of zeroes

$$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$= \frac{\text{Constant term}}{\text{Coefficient of } s^2}$$

Hence, relationship between the zeroes and the coefficients is true.

(iii) $p(x) = 6x^2 - 3 - 7x = 6x^2 - 7x - 3$

$= 6x^2 - 9x + 2x - 3$

$= 3x(2x - 3) + 1(2x - 3)$

$= (2x - 3)(3x + 1)$

For finding the zeroes put $p(x) = 0$

$(2x - 3)(3x + 1) = 0$

$x = \frac{3}{2}, \frac{-1}{3}$

Hence, zeroes of the polynomial are $\frac{3}{2}$ and $-\frac{1}{3}$.

Verification : Now sum of zeroes

$= \frac{3}{2} + \left(-\frac{1}{3}\right)$

$= \frac{9-2}{6} = \frac{7}{6}$

$= -\frac{(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $= \frac{3}{2} \times \frac{-1}{3} = \frac{-3}{6}$

$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(iv) $p(u) = 4u^2 + 8u$

$= 4u\,(u + 2)$

For finding the zeroes put $p(u) = 0$,

$4u\,(u + 2) = 0$

$u = 0, -2$

Hence, zeroes are 0 and – 2.

Verification :

Now sum of zeroes $= 0 + (-2)$

$= \frac{-2}{1} \times \frac{4}{4} = \frac{-8}{4}$

$= \frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$

Product of zeroes $= 0 \times (-2)$

$= \frac{0}{1} \times \frac{4}{4} = \frac{0}{4}$

$= \frac{\text{Constant term}}{\text{Coefficient of } u^2}$

Hence, relationship between the zeroes and the coefficients is true.

(v) $p(t) = t^2 - 15.$

$= t^2 - (\sqrt{15})^2$

$= (t - \sqrt{15})\,(t + \sqrt{15})$ $[\because a^2 - b^2 = (a + b)\,(a - b)]$

For finding the zeroes put $p(t) = 0$,

$(t - \sqrt{15})\,(t + \sqrt{15}) = 0$

$t = -\sqrt{15}, \sqrt{15}$

Verification :

Sum of the zeroes $= -\sqrt{15} + \sqrt{15} = \frac{0}{1}\cdot$

$= \frac{-(\text{Coefficient of } t)}{\text{Coefficient of } t^2}$

Product of zeroes $= -\sqrt{15} \times \sqrt{15} = -\frac{15}{1}$

$= \frac{\text{Constant term}}{\text{Coefficient of } t^2}$

Hence, relationship between the zeroes and the coefficients is true.

(vi) $p(x) = 3x^2 - x - 4$

$= 3x^2 - 4x + 3x - 4$

$= x(3x - 4) + 1\,(3x - 4)$

$= (x + 1)\,(3x - 4)$

For finding the zeroes put $p(x) = 0$,

$(x + 1)\,(3x - 4) = 0$

$x = -1, \frac{4}{3}$

Verification : Sum of zeroes $= -1 + \frac{4}{3} = \frac{1}{3}$

$= \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $= -1 \times \frac{4}{3} = \frac{-4}{3}$

$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Hence, relationship between the zeroes and the coefficients is true.

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4}, -1$ **(ii)** $\sqrt{2}, \frac{1}{3}$

(iii) $0, \sqrt{5}$ **(iv)** $1, 1$

(v) $-\frac{1}{4}, \frac{1}{4}$ **(vi)** $4, 1$

Sol. : **(i)** Let quadratic polynomial is $p(x)$ whose zeroes are α and β.

Given that : $\alpha + \beta = \frac{1}{4}$ and $\alpha\beta = -1$

So, required polynomial $= k[x^2 - (\alpha + \beta)x + \alpha\beta]$

$= k\left[x^2 - \frac{1}{4}x - 1\right]$

If $k = 4$, then $4x^2 - x - 4$.

(ii) Let quadratic polynomial is $ax^2 + bx + c$ whose zeroes are α and β.

Given that : $\alpha + \beta = \sqrt{2}$ and $\alpha\beta = \frac{1}{3}$

So, required polynomial $= k[x^2 - (\alpha + \beta)x + \alpha\beta]$

$= k\left[x^2 - \sqrt{2}x + \frac{1}{3}\right]$

For $k = 3$ quadratic polynomial is $3x^2 - 3\sqrt{2}x + 1$.

(iii) Let quadratic polynomial is $ax^2 + bx + c$ whose zeroes are α and β.

Given that : $\alpha + \beta = 0$ and $\alpha\beta = \sqrt{5}$

Hence, required polynomial $= k[x^2 - (\alpha + \beta)x + \alpha\beta]$

$= k[x^2 - 0 + \sqrt{5}]$

$= k(x^2 + \sqrt{5})$

For $k = 1$ quadratic polynomial is $x^2 + \sqrt{5}$.

(iv) Let quadratic polynomial is $ax^2 + bx + c$ zeroes are α and β.

Given that : $\alpha + \beta = 1$ and $\alpha\beta = 1$

Hence, required polynomial $= k[x^2 - (\alpha + \beta)x + \alpha\beta]$

$= k[x^2 - x + 1]$

For $k = 1$ quadratic polynomial is $x^2 - x + 1$.

(v) Let quadratic polynomial is $ax^2 + bx + c$ whose zeroes are α and β.

Given that : $\alpha + \beta = \frac{-1}{4}$ and $\alpha\beta = \frac{1}{4}$

So, required polynomial $= k[x^2 - (\alpha + \beta)x + \alpha\beta]$

$= k\left[x^2 + \frac{1}{4}x + \frac{1}{4}\right]$

For $k = 4$ quadratic polynomial is $4x^2 + x + 1$.

(vi) Let quadratic polynomial is $ax^2 + bx + c$ whose zeroes are α and β.

Given that : $\alpha + \beta = 4$ and $\alpha\beta = 1$

Hence, required polynomial $= x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - 4x + 1$

Pair of Linear Equations in Two Variables

3

NCERT SOLUTIONS

What's inside

– *In-Chapter Q's (solved)*

– *Textbook Exercise Q's (solved)*

Exercise – 3.1

1. Form the pair of linear equations in the following problems, and find their solutions graphically.

(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost ` 50, whereas 7 pencils and 5 pens together cost ` 46. Find the cost of one pencil and that of one pen.

2. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the line representing the following pairs of linear equations intersect at a point, are parallel or coincident :

(i) $5x - 4y + 8 = 0, 7x + 6y - 9 = 0$.

(ii) $9x + 3y + 12 = 0, 18x + 6y + 24 = 0$.

(iii) $6x - 3y + 10 = 0, 2x - y + 9 = 0$.

Sol. : **(i)** The given pair of linear equation are :

$$5x - 4y + 8 = 0 \quad ...(i)$$

$$7x + 6y - 9 = 0 \quad ...(ii)$$

Comparing the eqn. (i) and (ii) with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ respectively,

$$a_1 = 5, b_1 = -4, c_1 = 8$$

$$a_2 = 7, b_2 = 6, c_2 = -9$$

Here, $\frac{a_1}{a_2} = \frac{5}{7}, \frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3}$

$\because$ $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Thus, lines (i) and (ii) intersect at a point, that is, they are intersecting lines.

(ii) The given pair of linear equation are :

$$9x + 3y + 12 = 0 \quad ...(i)$$

$$18x + 6y + 24 = 0 \quad ...(ii)$$

Comparing the equation (i) and (ii) with the standard form of a pair linear equations,

$$a_1 = 9, b_1 = 3, c_1 = 12$$

$$a_2 = 18, b_2 = 6, c_2 = 24$$

Here, $\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}$,

$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$

$\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$

$\therefore$ $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, lines (i) and (ii) are coincident.

(iii) The given pair of linear equation are :

$$6x - 3y + 10 = 0 \quad ...(i)$$

$$2x - y + 9 = 0 \quad ...(ii)$$

Comparing the eqn. (i) and (ii) with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ respectively,

$$a_1 = 6, b_1 = -3, c_1 = 10$$
$$a_2 = 2, b_2 = -1, c_2 = 9$$

$$\frac{a_1}{a_2} = \frac{6}{2} = \frac{3}{1}, \frac{b_1}{b_2} = \frac{-3}{-1} = \frac{3}{1}, \frac{c_1}{c_2} = \frac{10}{9}$$

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, lines (i) and (ii) are parallel.

3. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.

(i) $3x + 2y = 5$; $2x - 3y = 7$.

(ii) $2x - 3y = 8$; $4x - 6y = 9$.

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$; $9x - 10y = 14$.

(iv) $5x - 3y = 11$; $-10x + 6y = -22$.

(v) $\frac{4}{3}x + 2y = 8$; $2x + 3y = 12$.

Sol. : (i) The given pair of linear equation are :

$$3x + 2y = 5 \quad ...(i)$$
$$2x - 3y = 7 \quad ...(ii)$$

Comparing the eqn. (i) and (ii) with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2x + c_2 = 0$ respectively, we get

$$a_1 = 3, b_1 = 2, c_1 = 5$$
$$a_2 = 2, b_2 = -3, c_2 = 7$$

Here, $\frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{-2}{3}, \frac{c_1}{c_2} = \frac{5}{7}$

$\because$ $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, pair of linear equations are consistent.

(ii) Given : $2x - 3y = 8$ ...(i)

$4x - 6y = 9$ ...(ii)

Comparing the eqn. (i) and (ii) with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ respectively,

$$a_1 = 2, b_1 = -3, c_1 = -8$$
$$a_2 = 4, b_2 = -6, c_2 = -9$$

Here, $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-8}{-9}$

$\because$ $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, lines are inconsistent.

(iii) The given pair of linear equation are :

$$\frac{3}{2}x + \frac{5}{3}y = 7$$

or $9x + 10y = 42$ ...(i)

$9x - 10y = 14$ ...(ii)

Comparing the eqn. (i) and (ii) with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ respectively, we get

$$a_1 = 9, b_1 = 10, c_1 = -42$$
$$a_2 = 9, b_2 = -10, c_2 = -14$$

Here, $\frac{a_1}{a_2} = \frac{9}{9} = \frac{1}{1}, \frac{b_1}{b_2} = \frac{10}{-10} = \frac{1}{-1}$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

∴ Lines are consistent.

(iv) The given pair of linear equation are :

$$5x - 3y = 11 \quad ...(i)$$
$$-10x + 6y = -22 \quad ...(ii)$$

Comparing the eqn. (i) and (ii) with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ respectively, we get

$$a_1 = 5, b_1 = -3, c_1 = -11$$
$$a_2 = -10, b_2 = 6, c_2 = +22$$

Here, $\frac{a_1}{a_2} = \frac{5}{-10} = \frac{1}{-2}, \frac{b_1}{b_2} = \frac{-3}{+6} = \frac{-1}{2},$

$$\frac{c_1}{c_2} = \frac{-11}{22} = \frac{-1}{2}$$

∴ $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

∴ The lines are consistent.

(v) The given pair of linear equation are :

$$\frac{4}{3}x + 2y = 8 \quad ...(i)$$
$$2x + 3y = 12 \quad ...(ii)$$

Comparing the eqn. (i) and (ii) with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ respectively, we get

∴ $a_1 = \frac{4}{3}, b_1 = 2, c_1 = -8$

$$a_2 = 2, b_2 = 3, c_1 = -12$$

$$\frac{a_1}{a_2} = \frac{\frac{4}{3}}{2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{2}{3},$$

$$\frac{c_1}{c_2} = \frac{-8}{-12} = \frac{2}{3}$$

∵ $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, lines are coincident.

4. Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically:

(i) $x + y = 5, 2x + 2y = 10$

(ii) $x - y = 8, 3x - 3y = 16$

(iii) $2x + y - 6 = 0, 4x - 2y - 4 = 0$

(iv) $2x - 2y - 2 = 0, 4x - 4y - 5 = 0$

5. The half perimeter of a rectangular garden is 36 m whose length is 4 m more than its breadth. Find the dimensions of garden.

Sol. : Let, length of the rectangular garden = l m

and breadth of the rectangular garden = b m

$\because$ Half perimeter of the garden = 36 m

$\therefore \quad l + b = 36 \quad$ ....(i)

and $\quad l - b = 4 \quad$ ....(ii)

Adding eqn. (i) and (ii),

$$\begin{aligned} l + b &= 36 \\ \underline{l - b} &\underline{= 4} \\ 2l &= 40 \\ l &= 20 \end{aligned}$$

Putting the value of l in eqn. (i),

$$\begin{aligned} 20 + b &= 36 \\ b &= 16 \end{aligned}$$

Hence, the dimensions of garden are 20 m and 16 m.

6. Given the linear equation 2x + 3y – 8 = 0, write another linear equation in two variables such that the geometrical representation of the pair so formed is:

(i) intersecting lines (ii) parallel lines

(iii) coincident lines

7. Draw the graphs of the equations x – y + 1 = 0 and 3x + 2y – 12 = 0. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

Exercise – 3.2

1. Solve the following pair of linear equations by the substitution method :

(i) $x + y = 14$, $x - y = 4$.

(ii) $s - t = 3$, $\frac{s}{3} + \frac{t}{2} = 6$.

(iii) $3x - y = 3$, $9x - 3y = 9$.

(iv) $0.2x + 0.3y = 1.3$, $0.4x + 0.5y = 2.3$.

(v) $\sqrt{2}x + \sqrt{3}y = 0$, $\sqrt{3}x - \sqrt{8}y = 0$

(vi) $\frac{3x}{2} - \frac{5y}{3} = -2$, $\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$.

Sol. : (i) Given :

$x + y = 14 \quad$ ...(i)

$x - y = 4 \quad$ ...(ii)

From eqn. (ii), $y = x - 4 \quad$ ...(iii)

Putting the value of y from eqn. (iii) in eqn. (i),

$$\begin{aligned} x + (x - 4) &= 14 \\ 2x &= 14 + 4 = 18 \end{aligned}$$

$$x = \frac{18}{2} = 9$$

Putting the value of x in eqn. (iii),

$$y = 9 - 4 = 5$$

So, $x = 9, y = 5.$

(ii) Given : $s - t = 3$ ...(i)

$$\frac{s}{3} + \frac{t}{2} = 6 \quad ...(ii)$$

From eqn. (i), $s = 3 + t$ ...(iii)

Putting the value of s from eqn. (iii) in eqn. (ii),

$$\frac{(3+t)}{3} + \frac{t}{2} = 6$$

$$1 + \frac{t}{3} + \frac{t}{2} = 6$$

$$\frac{5t}{6} = 5$$

$$t = 6$$

Putting the value of t in eqn. (iii),

$$s = 3 + 6 = 9$$

So, $s = 9, t = 6$

(iii) Given : $3x - y = 3$ ...(i)

$$9x - 3y = 9 \quad ...(ii)$$

From eqn. (i), $y = 3x - 3$ ...(iii)

Putting the value of y from eqn. (iii) in eqn. (ii),

$$9x - 3\,(3x - 3) = 9$$

$$9x - 9x + 9 = 9 \Rightarrow 9 = 9$$

Hence, both the given equations are coincide.

(iv) Given, $0.2x + 0.3y = 1.3$

$$0.4x + 0.5y = 2.3$$

We can also write them this way

$$\frac{2}{10}x + \frac{3}{10}y = \frac{13}{10}$$

$$2x + 3y = 13 \quad ...(i)$$

Similarly, $4x + 5y = 23$ ...(ii)

From eqn. (i), $2x = 13 - 3y$

$$x = \frac{13 - 3y}{2} \quad ...(iii)$$

Putting the value of x in eqn. (ii),

$$4\left(\frac{13 - 3y}{2}\right) + 5y = 23$$

$$26 - 6y + 5y = 23$$

$$-y = 23 - 26 = -3$$

$$y = 3$$

Putting the value of y in eqn. (iii),

$$x = \frac{13 - 3 \times 3}{2} = \frac{13 - 9}{2}$$

$$x = \frac{4}{2} = 2$$

Hence, $x = 2$ and $y = 3$

(v) Given, $\sqrt{2}x + \sqrt{3}y = 0$ ...(i)

$\sqrt{3}x - \sqrt{8}y = 0$ ...(ii)

From eqn. (ii), $x = \frac{\sqrt{8}}{\sqrt{3}}y$ ...(iii)

Putting the value of x from eqn. (iii) in eqn. (i),

$$\sqrt{2}\left(\frac{\sqrt{8}}{\sqrt{3}}y\right) + \sqrt{3}y = 0$$

$$4y + 3y = 0$$

$$7y = 0 \Rightarrow y = 0$$

Putting the value of y in eqn. (iii),

$$x = \frac{\sqrt{8}}{\sqrt{3}}(0) = 0$$

So, $x = 0, y = 0$

(vi) Given : $\frac{3x}{2} - \frac{5y}{3} = -2$

$$\frac{9x - 10y}{6} = -2$$

$9x - 10y = -12$ ...(i)

and $\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

$$\frac{2x + 3y}{6} = \frac{13}{6}$$

$2x + 3y = 13$ ...(ii)

From eqn. (ii), $2x = 13 - 3y$

$x = \frac{13 - 3y}{2}$ ...(iii)

Putting the value of x from eqn. (iii) in eqn. (i),

$$9\left(\frac{13 - 3y}{2}\right) - 10y = -12$$

$$\frac{117 - 27y}{2} - 10y = -12$$

$$117 - 27y - 20y = -24$$

$$-47y = -24 - 117$$

$$y = \frac{141}{47} = 3$$

Putting the value of y in eqn. (iii),

$$x = \frac{13 - 3(3)}{2}$$

$$= \frac{13 - 9}{2} = \frac{4}{2} = 2$$

Hence, $x = 2$ and $y = 3$.

2. Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$.

Sol. : Given, $2x + 3y = 11$ ...(i)

$2x - 4y = -24$ ...(ii)

From eqn. (ii), $2x = 4y - 24$

$x = 2y - 12$ ...(iii)

Putting the value of x from eqn. (iii) in eqn. (i),

$2(2y - 12) + 3y = 11$

$4y - 24 + 3y = 11$

$7y = 11 + 24 = 35$

$y = \frac{35}{7} = 5$

Putting the value of y in eqn. (iii),

$x = 2(5) - 12$

$= 10 - 12$

$x = -2$

So, $x = -2, y = 5$

Now, putting the value of x and y in $y = mx + 3$ $5 = m(-2) + 3$

$m = \frac{2}{-2} = -1$

$\therefore$ $m = -1$

3. Form the pair of linear equation for the following problems and find their solution by substitution method :

(i) The difference between two numbers is 26 and one number is three times the other find them.

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

(iii) The coach of a cricket team buys 7 bats and 6 balls for ₹3,800, Later, she buys 3 bats and 5 balls for ₹ 1,750. Find the cost of each bat and each ball.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is ₹ 105 and for a journey of 15 km, the charge paid is ₹ 155. What are the fixed charges and the charge per km ? How much does a person have to pay for travelling a distance of 25 km?

(v) A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator, if 3 is added to both the numerator and the denominator becomes $\frac{5}{6}$. Find the fraction.

(vi) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages ?

Sol. : **(i)** Let two numbers are x and y then

ATQ, $x - y = 26$ ...(i)

$x = 3y$ ...(ii)

Substituting the value of x from eqn. (ii) in eqn. (i),

$3y - y = 26$

$2y = 26$

$$y = \frac{26}{2} = 13$$

Putting the value of y in eqn. (ii),

$$x = 3(13) = 39$$

$\therefore \quad x = 39$ and $y = 13$

(ii) Let the two supplementary angles be $x°$ and $y°$, where $x < y$.

then $\quad x° + y° = 180° \quad$ ...(i)

$x° = y° - 18° \quad$ ...(ii)

Substituting the value of x from eqn. (ii) in eqn. (i), we get

$$(y° - 18°) + y° = 180°$$

$$2y° = 180° + 18°$$

$$y° = \frac{198°}{2} = 99°$$

Substituting the value of y in eqn. (ii), we get

$$x° = 99° - 18° = 81°$$

Thus, two supplementary angles are 81° and 99°.

(iii) Let the cost of a bat be ₹ x and that of a ball be ₹ y.

ATQ, $\quad 7x + 6y = 3800 \quad$ ...(i)

$3x + 5y = 1750 \quad$ ...(ii)

From eqn. (ii), $\quad 3x = 1750 - 5y$

$x = \frac{1750 - 5y}{3} \quad$ ...(iii)

Putting the value of x from eqn. (iii) in eqn. (i),

$$7\left(\frac{1750 - 5y}{3}\right) + 6y = 3800$$

$$12250 - 35y + 18y = 11400$$

$$-17y = 11400 - 12250$$

$$y = \frac{-850}{-17} = 50$$

Putting the value of y in eqn. (iii),

$$x = \frac{1750 - 250}{3}$$

$$= \frac{1500}{3} = 500$$

So, $\quad x = 500, y = 50$

Hence, cost of a bat = ₹ 500 and cost of a ball = ₹ 50.

(iv) Let the fixed charges of taxi be ₹ x and the charges per km be ₹ y.

then ATQ, $\quad x + 10y = 105 \quad$ ...(i)

and $\quad x + 15y = 155 \quad$ ...(ii)

From eqn. (i), $\quad x = 105 - 10y \quad$ ...(iii)

Putting the value of x from eqn. (iii) in eqn. (ii),

$$(105 - 10y) + 15y = 155$$

$$5y = 155 - 105$$

$$y = \frac{50}{5} = 10$$

Putting the value of y in eqn. (iii),

$$x = 105 - 10\,(10)$$
$$= 105 - 100 = 5$$

So, $x = 5, y = 10$

$\therefore$ Fixed charges of taxi = ₹ 5

Charges per km = ₹ 10

(v) Let numerator = x and denominator = y

$\therefore$ Fraction $= \frac{x}{y}$

ATQ, $\frac{x+2}{y+2} = \frac{9}{11}$

$$11x + 22 = 9y + 18$$
$$11x - 9y = -4 \quad \text{...(i)}$$

and $\frac{x+3}{y+3} = \frac{5}{6}$

$$6x + 18 = 5y + 15$$
$$6x - 5y = -3 \quad \text{...(ii)}$$

From eqn. (ii), $6x = 5y - 3$

$$x = \frac{5y-3}{6} \quad \text{...(iii)}$$

Putting the value of x from eqn. (iii) in eqn. (i),

$$11\left(\frac{5y-3}{6}\right) - 9y = -4$$
$$55y - 33 - 54y = -24$$
$$y = 9$$

Putting the value of y in eqn. (iii),

$$x = \frac{5(9)-3}{6} = \frac{45-3}{6} = \frac{42}{6}$$
$$x = 7$$

Hence, Fraction $= \frac{x}{y} = \frac{7}{9}$

(vi) Let, the present age of Jacob = x years and the present age of his son = y years, then after five years, Jacob's age = $x + 5$ and son's age = $y + 5$

ATQ, $x + 5 = 3(y + 5)$

$$x + 5 = 3y + 15$$
$$x - 3y = 10 \quad \text{...(i)}$$

Five years agoAge of Jacob $= x - 5$

and Age of son $= y - 5$

ATQ, $x - 5 = 7(y - 5)$

$$x - 5 = 7y - 35$$
$$x - 7y = -30 \quad \text{...(ii)}$$

From eqn. (i), $x = 10 + 3y$ ...(iii)

Putting the value of x from eqn. (iii) in eqn. (ii),

$$10 + 3y - 7y = -30$$
$$-4y = -30 - 10$$

$$= -40$$

$$y = \frac{40}{4} = 10$$

Putting the value of y in eqn. (i),

$$x - 3(10) = 10$$

$$x = 10 + 30 = 40$$

So, Age of Jacob = 40 year

and Age of Son = 10 year

Exercise – 3.3

1. Solve the following pair of linear equations by the elimination method :

(i) $x + y = 5, 2x - 3y = 4$.

(ii) $3x + 4y = 10, 2x - 2y = 2$.

(iii) $3x - 5y - 4 = 0, 9x = 2y + 7$.

(iv) $\frac{x}{2} + \frac{2y}{3} = -1, x - \frac{y}{3} = 3$

Sol. : (i) Given : $x + y = 5$ ...(i)

$2x - 3y = 4$ ...(ii)

Multiply by 3 in eqn. (i),

$3x + 3y = 15$ ...(iii)

Adding eqn. (ii) and (iii),

$$5x = 19$$

$$x = \frac{19}{5}$$

Putting the value of x in eqn. (i),

$$\frac{19}{5} + y = 5$$

$$y = 5 - \frac{19}{5} = \frac{25 - 19}{5}$$

$$y = \frac{6}{5}$$

So, $x = \frac{19}{5}, y = \frac{6}{5}$

(ii) $3x + 4y = 10$ ...(i)

$2x - 2y = 2$ ...(ii)

Multiplying by 2 in eqn. (ii),

$$4x - 4y = 4$$

$$\underline{3x + 4y = 10}$$

$$7x = 14$$

$$x = \frac{14}{7} = 2$$

Putting the value of x in eqn. (i),

$$3(2) + 4y = 10$$

$$4y = 10 - 6$$

$$y = \frac{4}{4} = 1$$

So, $x = 2, y = 1$

(iii) Given : $3x - 5y - 4 = 0$ ...(i)

$9x - 2y - 7 = 0$ ...(ii)

Multiplying by 3 in eqn. (i),

$9x - 15y - 12 = 0$ ...(iii)

$9x - 2y - 7 = 0$ ...(ii)

$\quad - \quad + \quad +$

Subtracting $-13y - 5 = 0$

$$-13y = 5$$

$$y = \frac{-5}{13}$$

Putting the value of y in eqn. (i),

$$3x - 5\left(\frac{-5}{13}\right) - 4 = 0$$

$$3x + \frac{25}{13} - 4 = 0$$

$$3x = \frac{27}{13}$$

$$x = \frac{9}{13}$$

So, $x = \frac{9}{13}, y = \frac{-5}{13}$

(iv) Given : $\frac{x}{2} + \frac{2y}{3} = -1$ ...(i)

and $x - \frac{y}{3} = 3$ ...(ii)

Multiplying eqn. (ii) by 2 and adding it to eqn. (i) we get,

$$2x - \frac{2y}{3} = 6$$

$$\frac{x}{2} + \frac{2y}{3} = -1$$

$$\frac{5x}{2} = 5$$

$$x = 2$$

Putting the value of x in eqn. (i),

$$\frac{2}{2} + \frac{2y}{3} = -1$$

$$\frac{2y}{3} = -2$$

$$y = -3$$

Hence, $x = 2, y = -3$.

2. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method :

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction ?

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu ?

(iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

(iv) Meena went to a bank to withdraw ₹2000. She asked the cashier to give her ₹50 and ₹100 notes only. Meena got 25 notes in all. Find how many notes of ₹ 50 and ₹ 100 she received.

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹ 27 for a book kept for seven days, while Susy paid ₹ 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Sol. : (i) Let, numerator be x and denominator be y, then

$$\text{Fraction} = \frac{x}{y}$$

ATQ, $\frac{x+1}{y-1} = 1$

$$x + 1 = y - 1$$

$$x - y = -2 \quad ...(i)$$

and $\frac{x}{y+1} = \frac{1}{2}$

$$2x = y + 1$$

$$2x - y = 1 \quad ...(ii)$$

Subtracting eqn. (i) from eqn. (ii),

$$2x - y = 1$$

$$x - y = -2$$

$$-\quad +\quad\quad +$$

$$x = 3$$

Putting the value of x in eqn. (i),

$$3 - y = -2$$

$$y = 5$$

So, fraction $= \frac{x}{y} = \frac{3}{5}$

(ii) Let Nuri's age = x years and Sonu's age = y years then five years ago,

Nuri's age = $x - 5$

Sonu's age = $y - 5$

ATQ, $x - 5 = 3(y - 5)$

$$x - 3y = -10 \quad ...(i)$$

Ten years later,

Nuri's age = $x + 10$

Sonu's age = $y + 10$

ATQ, $x + 10 = 2(y + 10)$

$$x + 10 = 2y + 20$$

$$x - 2y = 10 \quad ...(ii)$$

Subtracting eqn. (i) from eqn. (ii),

$$x - 2y = 10$$

$$x - 3y = -10$$

$$\underline{- \quad + \quad\quad +}$$

$y = 20$

Putting the value of y in eqn. (i),

$x - 3(20) = -10$

$x = -10 + 60$

$x = 50$

Hence, Nuri's age = 50 year

Sonu's age = 20 year

(iii) Let the unit digit of the number be y and the ten's digit be x then the number = $10x + y$

ATQ, $x + y = 9$ ...(i)

$9(10x + y) = 2(10y + x)$

[$\because$ Number after reversing the digits $10y + x$]

$90x + 9y = 20y + 2x$

$88x - 11y = 0$

$8x - y = 0$

$y = 8x$ ...(ii)

Putting the value of y in eqn. (i),

$x + 8x = 9$

$9x = 9$

$x = 1$

From eqn. (ii), $y = 8 \times 1 = 8$

Hence, Ten's digit = 1

Unit's digit = 8

Number = $10x + y = 10 \times 1 + 8 = 18$

(iv) Let the number of ₹ 50 notes and ₹ 100 notes be x and y respectively.

ATQ, $x + y = 25$ ...(i)

and $50x + 100y = 2000$

$x + 2y = 40$ ...(ii)

Subtracting eqn. (i) from eqn. (ii),

$x + 2y = 40$

$x + y = 25$

$$\underline{- \quad - \quad -}$$

$y = 15$

Putting the value of y in eqn. (i),

$x + 15 = 25$

$x = 25 - 15 = 10$

Hence, the number of ₹ 50 notes = 10

Number of ₹ 100 notes = 15

(v) Let the fixed charge for the first three days = ₹ x

and additional charge for each day = ₹ y, then

Sarita paid the charge to keep the book for 7 days.

= ₹ 27

$x + 4y = 27$ ...(i)

Rashi paid the charge for keeping the book for 5 days

= ₹ 21

$x + 2y = 21$ ...(ii)

Subtracting eqn. (ii) from eqn. (i),

$x + 4y = 27$

$x + 2y = 21$

$- \quad - \quad -$

$2y = 6$

$y = \frac{6}{2} = 3$

Putting the value of y in eqn. (i),

$x + 4(3) = 27$

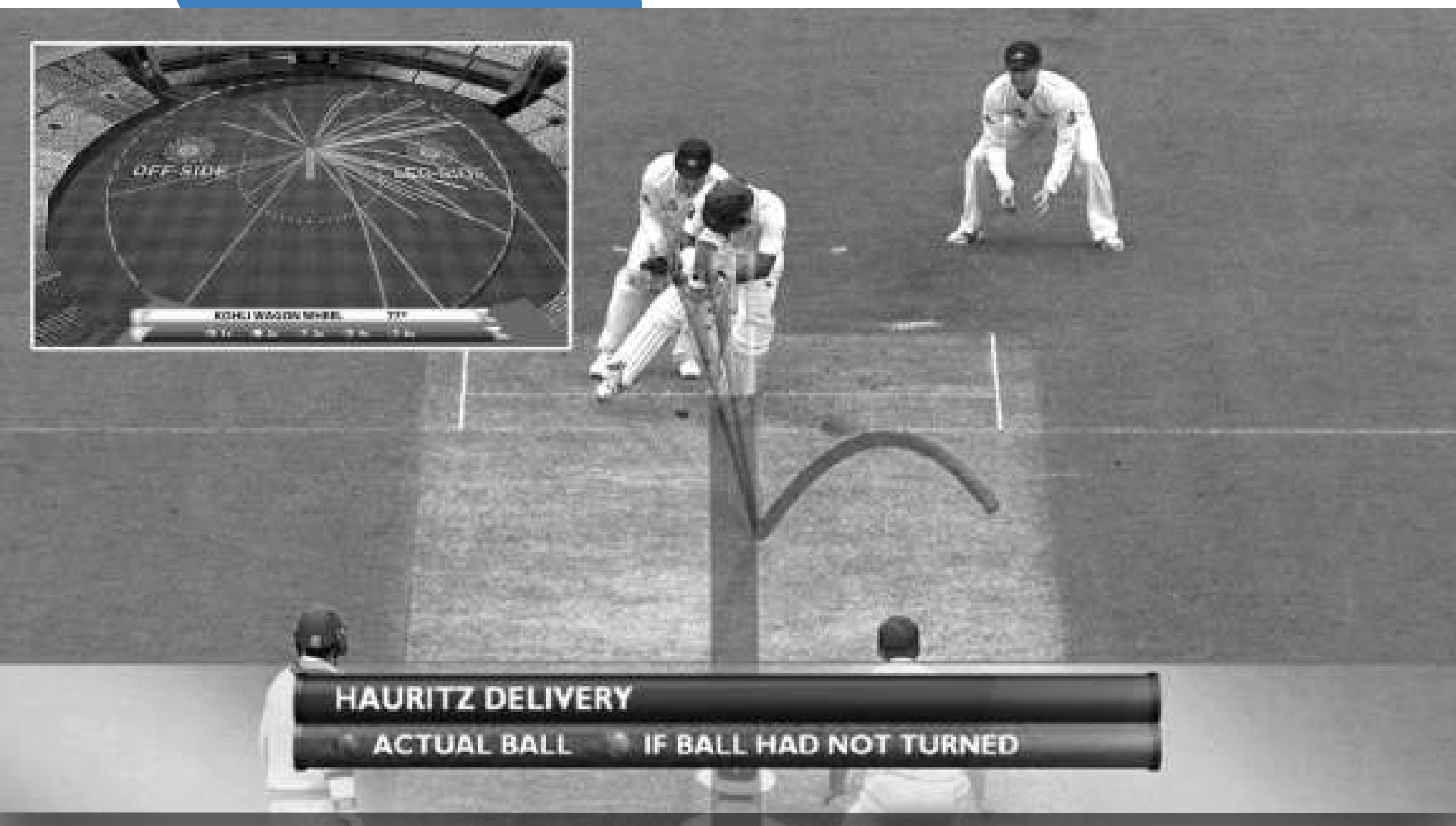
$x = 27 - 12 = 15$

Hence, the fixed charge for the first three days = ₹ 15

Additional charge for each day = ₹ 3

Quadratic Equations 4

NCERT SOLUTIONS



What's inside

– *In-Chapter Q's (solved)*
– *Textbook Exercise Q's (solved)*

Exercise – 4.1

1. Check whether the following are quadratic equation :

(i) $(x + 1)^2 = 2(x - 3)$

(ii) $x^2 - 2x = (-2)(3 - x)$

(iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$

(iv) $(x - 3)(2x + 1) = x(x + 5)$

(v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$

(vi) $x^2 + 3x + 1 = (x - 2)^2$

(vii) $(x + 2)^3 = 2x(x^2 - 1)$

(viii) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

Sol. : (i) Given : $(x + 1)^2 = 2(x - 3)$

$x^2 + 2x + 1 = 2x - 6$ $\quad [\because (a + b)^2 = a^2 + 2ab + b^2]$

$x^2 + 2x - 2x + 1 + 6 = 0$

$x^2 + 7 = 0$ ....(i)

Comparing equation (i) by $ax^2 + bx + c = 0$,

$a = 1, b = 0, c = 7$

Hence, the given equation is a quadratic equation because here the highest power of x is 2 and $a \neq 0$.

(ii) Given : $x^2 - 2x = -2(3 - x)$

$x^2 - 2x = -6 + 2x$

$x^2 - 2x - 2x + 6 = 0$

$x^2 - 4x + 6 = 0$ ...(i)

Comparing equation (i) by $ax^2 + bx + c = 0$,

$a = 1, b = -4, c = 6$

Hence, the given equation is a quadratic equation because here the highest power of x is 2 and $a \neq 0$.

(iii) Given :$(x - 2)(x + 1) = (x - 1)(x + 3)$

$x^2 - 2x + x - 2 = x^2 - x + 3x - 3$

$x^2 - x - 2 - x^2 - 2x + 3 = 0$

$-3x + 1 = 0$

or $3x - 1 = 0$ ...(i)

Comparing equation (i) by $ax^2 + bx + c = 0$,

$a = 0, b = 3, c = -1$

Hence, the given equation is not a quadratic equation because here the highest power of x is not 2 and $a = 0$.

(iv) Given :$(x - 3)(2x + 1) = x(x + 5)$

$2x^2 - 6x + x - 3 = x^2 + 5x$

$2x^2 - x^2 - 5x - 5x - 3 = 0$

$x^2 - 10x - 3 = 0$ ...(i)

Comparing equation (i) by $ax^2 + bx + c = 0$,

$a = 1, b = -10, c = -3$

Hence, the given equation is a quadratic equation because here the highest power of x is 2 and $a \neq 0$.

(v) Given : $(2x - 1)(x - 3) = (x + 5)(x - 1)$

$2x^2 - x - 6x + 3 = x^2 + 5x - x - 5$

$2x^2 - 7x + 3 = x^2 + 4x - 5$

$2x^2 - x^2 - 7x - 4x + 3 + 5 = 0$

$x^2 - 11x + 8 = 0$ ...(i)

Comparing equation (i) by $ax^2 + bx + c = 0$,

$a = 1, b = -11, c = 8$

Hence, the given equation is a quadratic equation because here the highest power of x is 2 and $a \neq 0$.

(vi) Given : $x^2 + 3x + 1 = (x - 2)^2$

$x^2 + 3x + 1 = x^2 - 4x + 4$

$x^2 - x^2 + 3x + 4x + 1 - 4 = 0$

$7x - 3 = 0$ ...(i)

Comparing equation (i) by $ax^2 + bx + c = 0$,

$a = 0, b = 7, c = -3$

Hence, the given equation is not a quadratic equation because here the highest power of x is not 2 and $a = 0$.

(vii) Given : $(x + 2)^3 = 2x(x^2 - 1)$

$x^3 + 8 + 6x^2 + 12x = 2x^3 - 2x$

$x^3 - 2x^3 + 6x^2 + 12x + 2x + 8 = 0$

$-x^3 + 6x^2 + 14x + 8 = 0$

Here, the maximum power of x is 3, so it is not a quadratic equation.

(viii) Given : $x^3 - 4x^2 - x + 1 = (x - 2)^3$

$x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x^2 + 12x$

$-4x^2 + 6x^2 - x - 12x + 1 + 8 = 0$

$2x^2 - 13x + 9 = 0$

Hence, the given equation is a quadratic equation because here the highest power of x is 2 and $a \neq 0$.

2. Represent the following situations in the form of quadratic equation :

(i) The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Sol. : (i) Given : The area of rectangular plot = 528 m^2

Let,breadth of rectangle = x meter, then

$$\text{Length} = 2x + 1$$

So, $$\text{Area} = \text{length} \times \text{breadth}$$

$$528 = x(2x + 1)$$

$$528 = 2x^2 + x$$

$$2x^2 + x - 528 = 0$$

(ii) Let, two consecutive positive integers are x and $(x + 1)$

A.T.Q., $x(x + 1) = 306x^2 + x$

$$x^2 + x = 306$$

$$x^2 + x - 306 = 0$$

(iii) Let, Age of Rohan = x year

and Age of mother = x + 26 year

After 3 years,

Age of Rohan = $(x + 3)$ year

Age of mother = $x + 26 + 3$

= $(x + 29)$ year

A.T.Q.,$(x + 3)(x + 29) = 360$

$$x^2 + 3x + 29x + 87 = 360$$

$$x^2 + 32x - 273 = 0$$

(iv) Let, Speed of train = x km/hr and Total distance = 480 km

then $$\text{Time} = \frac{\text{distance}}{\text{speed}} = \frac{480}{x}$$

A.T.Q., $$\frac{480}{x-8} - \frac{480}{x} = 3$$

$$480(x - x + 8) = 3[x(x - 8)]$$

$$480(8) = 3[x^2 - 8x]$$

$$3x^2 - 24x - 3840 = 0$$

$$3[x^2 - 8x - 1280] = 0$$

$$x^2 - 8x - 1280 = 0$$

Exercise – 4.2

1. Find the roots of the following quadratic equations by factorisation :

(i) $x^2 - 3x - 10 = 0$

(ii) $2x^2 + x - 6 = 0$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

(iv) $2x^2 - x + \frac{1}{8} = 0$

(v) $100x^2 - 20x + 1 = 0$

Sol. : **(i)** Given : $x^2 - 3x - 10 = 0$

$x^2 - 5x + 2x - 10 = 0$

$x(x - 5) + 2(x - 5) = 0$

$(x - 5)(x + 2) = 0$

If $x - 5 = 0 \Rightarrow x = 5$

If $x + 2 = 0 \Rightarrow x = -2$

Hence, the roots of the equation $x^2 - 3x - 10$ are -2 and 5.

(ii) Given : $2x^2 + x - 6 = 0$

$2x^2 + 4x - 3x - 6 = 0$

$2x(x + 2) - 3(x + 2) = 0$

$(x + 2)(2x - 3) = 0$

If $x + 2 = 0 \Rightarrow x = -2$

If $2x - 3 = 0 \Rightarrow x = \frac{3}{2}$

Hence, the roots of the equation $2x^2 + x - 6 = 0$ are -2 and $\frac{3}{2}$.

(iii) Given : $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$

$x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$

$(\sqrt{2}x + 5)(x + \sqrt{2}) = 0$

If $\sqrt{2}x + 5 = 0 \Rightarrow x = -\frac{5}{\sqrt{2}}$

If $x + \sqrt{2} = 0 \Rightarrow x = -\sqrt{2}$

Hence, the roots of the equation $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ are $\frac{-5}{\sqrt{2}}$ and $-\sqrt{2}$.

(iv) Given : $2x^2 - x + \frac{1}{8} = 0$

Multiplying the equation by 8, we get :

$16x^2 - 8x + 1 = 0$

$16x^2 - 4x - 4x + 1 = 0$

$4x(4x - 1) - 1(4x - 1) = 0$

$(4x - 1)(4x - 1) = 0$

$x = \frac{1}{4}, \frac{1}{4}$

Hence, the roots of the equation $2x^2 - x + \frac{1}{8} = 0$ are $\frac{1}{4}$ and $\frac{1}{4}$.

(v) Given : $100x^2 - 20x + 1 = 0$

$100x^2 - 10x - 10x + 1 = 0$

$10x(10x - 1) - 1(10x - 1) = 0$

$(10x - 1)(10x - 1) = 0$

$x = \frac{1}{10}, \frac{1}{10}$

Hence, the roots of the equation $100x^2 - 20x + 1$ are $\frac{1}{10}$ and $\frac{1}{10}$.

2. Solve the problems given in Example 1.

3. Find two numbers whose sum is 27 and product is 182.

Sol. : Let two numbers be x and y, then

A.T.Q., $x + y = 27$ ...(i)

$xy = 182$ ...(ii)

From eqn. (i), $x = 27 - y$ ...(iii)

Put the value of x from eqn. (iii) in eqn. (ii), we get

$(27 - y)y = 182$

$27y - y^2 = 182$

$y^2 - 27y + 182 = 0$

$y^2 - (14 + 13)y + 182 = 0$

$y^2 - 14y - 13y + 182 = 0$

$y(y - 14) - 13(y - 14) = 0$

$(y - 14)(y - 13) = 0$

If $y - 14 = 0$ then $y = 14$

If $y - 13 = 0$ then $y = 13$

In eqn. (iii), if $y = 14$ then $x = 27 - 14 = 13$

If $y = 13$ then $x = 27 - 13 = 14$

Hence, numbers are 13 and 14.

4. Find two consecutive positive integers, sum of whose squares is 365.

Sol. : Let x and $x + 1$ be two consecutive positive integer.

Then A.T.Q., $x^2 + (x + 1)^2 = 365$

$x^2 + x^2 + 2x + 1 = 365$ $[\because (a + b)^2 = a^2 + 2ab + b^2]$

$2x^2 + 2x - 364 = 0$

$2(x^2 + x - 182) = 0$

$x^2 + x - 182 = 0$

$x^2 + (14 - 13)x - 182 = 0$

$x^2 + 14x - 13x - 182 = 0$

$x(x + 14) - 13(x + 14) = 0$

$(x + 14)(x - 13) = 0$

If $x + 14 = 0 \Rightarrow x = -14$

and $x - 13 = 0 \Rightarrow x = 13$

$\because$ x is a positive integer.

$\Rightarrow x = -14$ is invalid.

So, $x = 13$ and $x + 1 = 13 + 1 = 14$

Hence, two consecutive positive integers are 13 and 14.

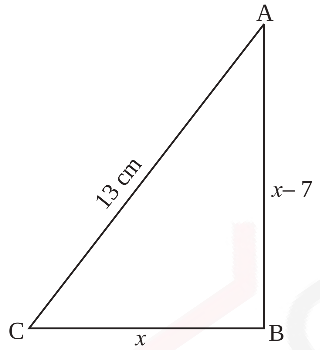
5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find other two sides.

Sol. : Given : Hypotenuse of a right angled triangle = 13 cm

Let, base = x cm, then height = $(x - 7)$ cm

$\Rightarrow$ By pythagoras theorem in ABC

$$AC^2 = BC^2 + AB^2$$



$$13^2 = x^2 + (x - 7)^2$$

$$169 = x^2 + x^2 - 14x + 49$$

$$2x^2 - 14x - 120 = 0$$

$$x^2 - 7x - 60 = 0$$

$$x^2 - (12 - 5)x - 60 = 0$$

$$x^2 - 12x + 5x - 60 = 0$$

$$x(x - 12) + 5(x - 12) = 0$$

$$(x - 12)(x + 5) = 0$$

If $x - 12 = 0 \Rightarrow x = 12$

and $x + 5 = 0 \Rightarrow x = -5$

Hence, $x = 12$ or -5

Because base cannot be negative.

$\therefore$ $x = 12$

Thus, Base $(BC) = 12$ cm

Height $(AB) = 12 - 7 = 5$ cm

6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ₹ 90, find the number of articles produced and the cost of each article.

Sol. : Let, number of articles produced $= x$

and,the cost of each article $= 2x + 3$

So, Total cost of production = Number of articles × cost of each article

$= x(2x + 3)$

A.T.Q., $x(2x + 3) = 90$

$2x^2 + 3x - 90 = 0$

$2x^2 + (15 - 12)x - 90 = 0$

$2x^2 + 15x - 12x - 90 = 0$

$x(2x + 15) - 6(2x + 15) = 0$

$(2x + 15)(x - 6) = 0$

If $2x + 15 = 0 \Rightarrow x = -\frac{15}{2}$

and $x - 6 = 0 \Rightarrow x = 6$

Number of articles cannot be negative.

$\Rightarrow$ $x = -\frac{15}{2}$

So, $x = 6$

and $2x + 3 = 15$

Hence, number of articles produced is 6 and the cost of each article is ₹ 15.

Arithmetic Progressions 5

NCERT SOLUTIONS

What's inside

– In-Chapter Q's (solved)

– Textbook Exercise Q's (solved)

Exercise – 5.1

1. **In which of the following situations, does the list of numbers involved make an arithmetic progression, and why ?**

   (i) The taxi fare after each km when the fare is ₹ 15 for the first km and ₹ 8 for each additional km.

   (ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

   (iii) The cost of digging a well after every metre of digging, when it costs ₹ 150 for the first metre and rises by ₹ 50 for each subsequent metre.

   (iv) The amount of money in the account every year, when ₹ 10,000 is deposited at compound interest at 8% per annum.

Sol. : (i) According to the question, the fare for the first km *i.e.*, 1 km is ₹ 15. The fare for the next 2nd, 3rd, 4th km is ₹(15 + 8), ₹(15 + 2 × 8) and ₹(15 + 3 × 8) respectively.

i.e., 15, 23, 31, 39, ...

Here, each next term is obtained by adding 8 to the previous term.

Hence, it is an arithmetic progression.

(ii) Let the amount of air present in the cylinder is x then according to the question, the list obtained is as follow :

$$x, \left(x - \frac{x}{4} = \frac{3x}{4}\right), \left(\frac{3x}{4} - \frac{1}{4} \times \frac{3x}{4} = \frac{12x - 3x}{16} = \frac{9x}{16}\right), \ldots\ldots\ldots\ldots$$

or, $x, \frac{3x}{4}, \frac{9x}{16}, \ldots\ldots\ldots\ldots\ldots$

Now, $d = a_2 - a_1$

$$= \frac{3x}{4} - x = \frac{3x - 4x}{4} = \frac{-x}{4}$$

and $d = a_3 - a_2$

$$= \frac{9x}{16} - \frac{3x}{4} = \frac{9x - 12x}{16} = \frac{-3x}{16}$$

Here, $a_2 - a_1 \neq a_3 - a_2$

Hence, the above series is not an A.P. because common difference is not equal.

(iii) According to question,

The cost of 1 metre digging = ₹ 150

The cost of second metre digging

= ₹(150 + 50)

= ₹ 200

Thus, the cost of digging the 3rd, 4th, 5th metre are ₹ (200 + 50), (250 + 50), (300 + 50) respectively.

i.e., 150, 200, 250, 300, 350, ...

Here, each next term is obtained by adding 50 to the previous term.

Hence, it is an arithmetic progression.

(iv) According to the question, the amount in the account in the first year, second year, third year, fourth year,....

$10,000, \left[10,000\left(1+\frac{8}{100}\right)\right], \left[10,000\left(1+\frac{8}{100}\right)^2\right],$ .... respectively

or $10,000,\ 10,000\times\frac{108}{100},\ 10,000\times\frac{108}{100}\times\frac{108}{100},$ ....

or 10,000, 10,800, 11,664, ...

Now, $d = a_2 - a_1 = 10,800 - 10,000 = 800$

and $d = a_3 - a_2 = 11,664 - 10,800 = 864$

Here, $a_2 - a_1 \neq a_3 - a_1$

Hence, the above series is not an A.P.

2. **Write first four terms of the A. P., when the first term *a* and the common difference *d* are given as follows :**

(i) $a = 10, d = 10$ (ii) $a = -2, d = 0$

(iii) $a = 4, d = -3$ (iv) $a = -1, d = \frac{1}{2}$

(v) $a = -1.25, d = -0.25$

Sol. : (i) Given : First term $(a) = 10$

and Common difference $(d) = 10$

Let, the first four terms of arithmetic progression are $a, a + d, a + 2d, a + 3d$.

then $a = 10$

$a + d = 10 + 10 = 20$

$a + 2d = 10 + 20 = 30$

$a + 3d = 10 + 30 = 40$

Hence, the first four terms of the arithmetic progression are 10, 20, 30, 40.

(ii) Given : First term $(a) = -2$

and Common difference $(d) = 0$

Let, the first four terms of arithmetic progression are $a, a + d, a + 2d$ and $a + 3d$.

then $a = -2$

$a + d = -2 + 0 = -2$

$a + 2d = -2 + 2(0) = -2$

$a + 3d = -2 + 3(0) = -2$

Hence, the first four terms of the arithmetic progression are – 2, – 2, – 2, – 2.

(iii) Given :First term $(a) = 4$

andCommon difference $(d) = -3$

Let, the first four terms of arithmetic progression are $a, a + d, a + 2d, a + 3d$.

$a = 4$

$a + d = 4 + (-3) = 4 - 3 = 1$

$a + 2d = 4 + 2(-3) = 4 - 6 = -2$

$a + 3d = 4 + 3(-3) = 4 - 9 = -5$

Hence, the first four terms of arithmetic progression are 4, 1, – 2, – 5.

(iv) Given : First term $(a) = -1$

and Common difference $(d) = \frac{1}{2}$

then, first four terms of arithmetic progression

$a = -1$

$a + d = -1+\frac{1}{2} = -\frac{1}{2}$

$a + 2d = -1+2\left(\frac{1}{2}\right) = -1 + 1 = 0$

$a + 3d = -1+3\left(\frac{1}{2}\right) = -1+\frac{3}{2} = \frac{1}{2}$

Hence, the first four terms of arithmetic progression are – 1, $-\frac{1}{2}, 0, \frac{1}{2}$.

(v) Given : First term (a) = – 1.25

andCommon difference (d) = – 0.25

then, first four terms of arithmetic progression

$a = -1.25$

$a + d = -1.25 + (-0.25) = -1.25 - 0.25$

$= -1.50$

$a + 2d = -1.25 + 2(-0.25) = -1.25 - 0.50$

$= -1.75$

$a + 3d = -1.25 + 3(-0.25) = -1.25 - 0.75$

$= -2.00$

Hence, the first four terms of arithmetic progression are – 1.25, – 1.50, – 1.75 and – 2.00.

3. **For the following APs, write First term and the common difference :**

(i) 3, 1, – 1, – 3, ...

(ii) – 5, – 1, 3, 7, ...

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, ...$

(iv) 0.6, 1.7, 2.8, 3.9, ...

Sol. : (i) Given AP : 3, 1, – 1, – 3 ...

First term (a) = 3

Common difference (d) = 1 – 3 = – 2

(ii) Given AP : – 5, – 1, 3, 7 ...

First term (a) = – 5

Common difference (d) $= a_2 - a_1 = -1 - (-5)$

$= -1 + 5 = 4$

(iii) Given AP : $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}$, ...

First term (a) = $\frac{1}{3}$

Common difference (d) $= a_2 - a_1 = \frac{5}{3}-\frac{1}{3} = \frac{4}{3}$

(iv) Given AP :

0.6, 1.7, 2.8, 3.9 ....

First term (a) = 0.6

Common difference (d) $= a_2 - a_1 = 1.7 - 0.6$

$= 1.1$

4. Which of the following are APs ? If they form an A. P. find the common difference *d* and write three more terms.

(i) 2, 4, 8, 16, ...

(ii) $2, \frac{5}{2}, 3, \frac{7}{2}, ...$

(iii) – 1.2, – 3.2, – 5.2, – 7.2, ...

(iv) – 10, – 6, – 2, 2, ...

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, ...$

(vi) 0.2, 0.22, 0.222, 0.2222, ...

(vii) 0, – 4, – 8, – 12, ...

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, ...$

(ix) 1, 3, 9, 27, ...

(x) $a, 2a, 3a, 4a, ...$

(xi) $a, a^2, a^3, a^4, ...$

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, ...$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, ...$

(xiv) $1^2, 3^2, 5^2, 7^2, ...$

(xv) $1^2, 5^2, 7^2, 73, ...$

Sol. : (i) 2, 4, 8, 16, ...

Here, $a_1 = 2, a_2 = 4, a_3 = 8, a_4 = 16$

$a_2 - a_1 = 4 - 2 = 2$

$a_3 - a_2 = 8 - 4 = 4$

$a_4 - a_3 = 16 - 8 = 8$

It is clear that, $a_2 - a_1 \neq a_3 - a_2 \neq a_4 - a_3$

Hence, the given series is not A.P., because common difference is not equal.

(ii) Given series : $2, \frac{5}{2}, 3, \frac{7}{2}, ...$

Here, $a_1 = 2, a_2 = \frac{5}{2}, a_3 = 3, a_4 = \frac{7}{2}$

$a_2 - a_1 = \frac{5}{2} - 2 = \frac{5-4}{2} = \frac{1}{2}$

$a_3 - a_2 = 3 - \frac{5}{2} = \frac{6-5}{2} = \frac{1}{2}$

$a_4 - a_3 = \frac{7}{2} - 3 = \frac{7-6}{2} = \frac{1}{2}$

Hence, the given series is an A.P. because here the common difference $d = \frac{1}{2}$ is equal.

Clearly the next term will be obtained by adding $\frac{1}{2}$ to the previous term.

Hence, $a_5 = \frac{7}{2} + \frac{1}{2} = \frac{8}{2} = 4$

$a_6 = 4 + \frac{1}{2} = \frac{8+1}{2} = \frac{9}{2}$

$a_7 = \frac{9}{2} + \frac{1}{2} = \frac{10}{2} = 5$

(iii) Given series :

$$-1.2, -3.2, -5.2, -7.2 ...$$

Here, $a_1 = -1.2, a_2 = -3.2, a_3 = -5.2, a_4 = -7.2$

$a_2 - a_1 = -3.2 - (-1.2) = -3.2 + 1.2 = -2$

$a_3 - a_2 = -5.2 - (-3.2) = -5.2 + 3.2 = -2$
$a_4 - a_3 = -7.2 - (-5.2) = -7.2 + 5.2 = -2$

Hence, here the common difference $d = -2$ is equal. So, the above series is an A.P.

Clearly the next term will be obtained by adding –2 to the previous term.

$a_5 = -7.2 + (-2) = -7.2 - 2 = -9.2$
$a_6 = -9.2 + (-2) = -9.2 - 2 = -11.2$
$a_7 = -11.2 + (-2) = -11.2 - 2 = -13.2$

(iv) Given series :

$$-10, -6, -2, 2 \ldots..$$

Here, $a_1 = -10, a_2 = -6, a_3 = -2, a_4 = 2$
$a_2 - a_1 = -6 - (-10) = -6 + 10 = 4$
$a_3 - a_2 = -2 - (-6) = -2 + 6 = 4$
$a_4 - a_3 = 2 - (-2) = 2 + 2 = 4$

It is clear that, here the common difference $d = 4$ is the same. So, the above is an A.P.
Now, the next term

$a_5 = a_4 + d = 2 + 4 = 6$
$a_6 = a_5 + d = 6 + 4 = 10$
$a_7 = a_6 + d = 10 + 4 = 14$

(v) Given series :

$$3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$$

Here, $a_1 = 3$, $a_2 = 3+\sqrt{2}$, $a_3 = 3+2\sqrt{2}$, $a_4 = 3+3\sqrt{2}$

$a_2 - a_1 = 3+\sqrt{2}-3 = \sqrt{2}$

$a_3 - a_2 = 3+2\sqrt{2}-(3+\sqrt{2})$

$= 3+2\sqrt{2}-3-\sqrt{2} = \sqrt{2}$

$a_4 - a_3 = 3+3\sqrt{2}-(3+2\sqrt{2})$

$= 3+3\sqrt{2}-3-2\sqrt{2} = \sqrt{2}$

It is clear that here the common difference $d = \sqrt{2}$ is same. So the above is an A.P.
Now, the next term

$a_5 = a_4 + d = 3+3\sqrt{2}+\sqrt{2}$

$= 3+4\sqrt{2}$

$a_6 = a_5 + d = 3+4\sqrt{2}+\sqrt{2}$

$= 3+5\sqrt{2}$

$a_7 = a_6 + d = 3+5\sqrt{2}+\sqrt{2}$

$= 3+6\sqrt{2}$

(vi) Given series :

$$0.2, 0.22, 0.222, 0.2222 \ldots$$

Here, $a_1 = 0.2, a_2 = 0.22, a_3 = 0.222, a_4 = 0.2222$
$a_2 - a_1 = 0.22 - 0.2 = 0.02$

$a_3 - a_2 = 0.222 - 0.22 = 0.002$

Here, the common difference (d) is not same because :

$a_2 - a_1 \neq a_3 - a_2$

So, the above series is not an A.P.

(vii) Given series : 0, – 4, – 8, – 12 ...

Here, $a_1 = 0, a_2 = -4, a_3 = -8, a_4 = -12$

$a_2 - a_1 = -4 - 0 = -4$

$a_3 - a_2 = -8 - (-4) = -8 + 4 = -4$

$a_4 - a_3 = -12 - (-8) = -12 + 8 = -4$

Here, the common difference (d) = – 4 is same, so the above is an A.P.

Now, the next term

$a_5 = a_4 + d = -12 + (-4)$

$= -12 - 4 = -16$

$a_6 = a_5 + d = -16 + (-4)$

$= -16 - 4 = -20$

$a_7 = a_6 + d = -20 + (-4)$

$= -20 - 4 = -24$

(viii) Given series : $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, ...$

Here, $a_1 = -\frac{1}{2}, a_2 = -\frac{1}{2}, a_3 = -\frac{1}{2}, a_4 = -\frac{1}{2}$

$a_2 - a_1 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$

$a_3 - a_2 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$

$a_4 - a_3 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$

$\therefore$ Here, common difference (d) = 0 is same, so the above is an A.P.

Now, the next term

$a_5 = a_4 + d = -\frac{1}{2} + 0 = -\frac{1}{2}$

$a_6 = a_5 + d = -\frac{1}{2} + 0 = -\frac{1}{2}$

$a_7 = a_6 + d = -\frac{1}{2} + 0 = -\frac{1}{2}$

(ix) Given : 1, 3, 9, 27 ...

Here, $a_1 = 1, a_2 = 3, a_3 = 9, a_4 = 27$

$a_2 - a_1 = 3 - 1 = 2$

$a_3 - a_2 = 9 - 3 = 6$

$\because \quad a_2 - a_1 \neq a_3 - a_2$

Here, the common difference (d) is not same. So, the above is not A.P.

(x) Given series : $a, 2a, 3a, 4a, ...$

Here, $a_1 = a, a_2 = 2a, a_3 = 3a, a_4 = 4a$

$a_2 - a_1 = 2a - a = a$

$a_3 - a_2 = 3a - 2a = a$

$a_4 - a_3 = 4a - 3a = a$

Here, the common difference (d) = a is not same. So, the above is an A.P.

Now, the next term

$a_5 = a_4 + d = 4a + a = 5a$

$a_6 = a_5 + d = 5a + a = 6a$

$a_7 = a_6 + d = 6a + a = 7a$

(xi) Given : a, a^2, a^3, a^4 ...

Here, $a_1 = a, a_2 = a^2, a_3 = a^3, a_4 = a^4$

$a_2 - a_1 = a^2 - a$

$a_3 - a_2 = a^3 - a^2$

$\because \quad a_2 - a_1 \neq a_3 - a_1$

Hence, the given series is not an A.P. because the common difference (d) is not equal.

(xii) Given : $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, ...$

Here, $a_1 = \sqrt{2}$, $a_2 = \sqrt{8} = 2\sqrt{2}$, $a_3 = \sqrt{18} = 3\sqrt{2}$,

$a_4 = \sqrt{32} = 4\sqrt{2}$

$a_2 - a_1 = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

$a_3 - a_2 = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

$a_4 - a_3 = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

$\therefore$ Here, the common difference (d) = $\sqrt{2}$ is equal.So, the above series is an A.P.

Next three terms,

$a_5 = a_4 + d = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50}$

$a_6 = a_5 + d = 5\sqrt{2} + \sqrt{2} = 6\sqrt{2} = \sqrt{72}$

$a_7 = a_6 + d = 6\sqrt{2} + \sqrt{2} = 7\sqrt{2} = \sqrt{98}$

(xiii) Given : $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, ...$

Here, $a_1 = \sqrt{3}$, $a_2 = \sqrt{6}$, $a_3 = \sqrt{9}$, $a_4 = \sqrt{12}$

$a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{3}(\sqrt{2} - 1)$

$a_3 - a_2 = \sqrt{9} - \sqrt{6} = \sqrt{3}(\sqrt{3} - \sqrt{2})$

$\because \quad a_2 - a_1 \neq a_3 - a_2$

Hence, the given series is not an A.P. because the common difference is not equal.

(xiv) Given : $1^2, 3^2, 5^2, 7^2, ...$

Here, $a_1 = 1^2 = 1, a_2 = 3^2 = 9, a_3 = 5^2 = 25, a_4 = 7^2 = 49$

$a_2 - a_1 = 9 - 1 = 8$

$a_3 - a_2 = 25 - 9 = 16$

$\because \quad a_2 - a_1 \neq a_3 - a_2$

Hence, the given series is not an A.P. because the common difference is not equal.

(xv) Given : $1^2, 5^2, 7^2, 73, ...$

Here, $a_1 = 1^2 = 1, a_2 = 5^2 = 25, a_3 = 7^2 = 49, a_4 = 73$

$a_2 - a_1 = 25 - 1 = 24$

$a_3 - a_2 = 49 - 25 = 24$

$a_4 - a_3 = 73 - 49 = 24$

$\therefore$ Here, the common difference (d) = 24 is equal. So, the above series is an A.P.

Next three terms

$a_5 = a_4 + d = 73 + 24 = 97$

$a_6 = a_5 + d = 97 + 24 = 121$

$a_7 = a_6 + d = 121 + 24 = 145$

Exercise – 5.2

1. Fill in the blanks in the following table, given that *a* is the First term, *d* the common difference and a_n the n^{th} term of the A. P. :

	a	d	n	a_n
(i)	7	3	8	......
(ii)	– 18	......	10	0
(iii)	......	– 3	18	– 5
(iv)	– 18.9	2.5	......	3.6
(v)	3.5	0	105	......

Sol. : (i) n^{th} term of A.P.

$a_n = a + (n - 1)d$

$= 7 + (8 - 1) \times 3 = 7 + 7 \times 3$

$= 7 + 21 = 28$

(ii) n^{th} term of A.P.

$a_n = a + (n - 1)d$

$0 = -18 + (10 - 1)d$

$18 = 9d$

$d = \frac{18}{9} = 2$

(iii) n^{th} term of A.P.

$a_n = a + (n - 1)d$

$-5 = a + (18 - 1)(-3)$

$-5 = a - 51$

$a = 51 - 5 = 46$

(iv) n^{th} term of A.P.

$a_n = a + (n - 1)d$

$3.6 = -18.9 + (n - 1)(2.5)$

$$3.6 + 18.9 = (n - 1)(2.5)$$
$$22.5 = (n - 1)(2.5)$$
$$(n - 1) = \frac{22.5}{2.5} = 9$$
$$n - 1 = 9$$
$$n = 10$$

(v) n^{th} term of A.P.

$$a_n = a + (n - 1)d$$
$$= 3.5 + (105 - 1) \times 0$$
$$= 3.5 + 0 = 3.5$$

2. Choose the correct choice in the following and justify :

(i) 30^{th} term of the A.P. : 10, 7, 4, ... is :

(a) 97 (b) 77

(c) – 77 (d) – 87

(ii) 11^{th} term of the A.P. $-3, -\frac{1}{2}, 2, \ldots$ is :

(a) 28 (b) 22

(c) –38 (d) $-48\frac{1}{2}$

Sol. : (i) (c) A.P. 10, 7, 4, ...

$a = 10$, $d = 7 - 10 = -3$, $n = 30$

then, nth term $a_n = a + (n - 1)d$

$$a_{30} = 10 + (30 - 1)(-3)$$
$$a_{30} = 10 - 87 = -77$$

(ii) (b) $-3, -\frac{1}{2}, 2, \ldots$

$$a = -3,\ d = -\frac{1}{2} - (-3) = -\frac{1}{2} + 3$$
$$= \frac{-1+6}{2} = \frac{5}{2}$$

$\because$ $a_n = a + (n - 1)d$

$$\therefore \quad a_{11} = -3 + (11 - 1)\frac{5}{2}$$
$$= -3 + 10 \times \frac{5}{2}$$
$$= -3 + 25 = 22$$

3. In the following A. P. find the missing terms in the boxes :

(i) 2, □, 26

(ii) □, 13, □, 3.

(iii) 5, □, □, $9\frac{1}{2}$.

(iv) – 4, □, □, □, □, 6.

(v) □, 38, □, □, □, – 22.

Sol. : (i) Let 2, □, 26 are the terms of A.P. a, $a + d$, $a + 2d$ respectively.

$\therefore \quad a = 2$

and $\quad a + 2d = 26$

$2 + 2d = 26$

$2d = 26 - 2 = 24$

$d = \frac{24}{2} = 12$

Hence, missing term = $a + d = 2 + 12 = 14$.

(ii) Let □, 13, □, 3 are the terms of A.P. $a, a + d, a + 2d, a + 3d$ respectively.

then $\quad a + d = 13 \quad$ ...(i)

$a + 3d = 3 \quad$ ...(ii)

Subtracting eqn. (i) from eqn. (ii), we get

$$\begin{array}{r} a + 3d = 3 \\ a + d = 13 \\ - \quad - \quad - \\ \hline 2d = -10 \end{array}$$

$d = \frac{-10}{2} = -5$

Put the value of d in eqn. (i),

$a + (-5) = 13$

$a = 13 + 5 = 18$

Hence, missing term $\quad a = 18$

and $\quad a + 2d = 18 + 2(-5)$

$= 18 - 10 = 8$

(iii) Let 5, □, □, $9\frac{1}{2}$ are the terms of A.P. $a, a + d, a + 2d, a + 3d$ respectively.

then $\quad a = 5$

and $\quad a + 3d = \frac{19}{2}$

$5 + 3d = \frac{19}{2}$

$3d = \frac{19}{2} - 5 = \frac{19-10}{2} = \frac{9}{2}$

$d = \frac{9}{2 \times 3} = \frac{3}{2}$

Hence, missing term

$a + d = 5 + \frac{3}{2} = \frac{13}{2}$

and $\quad a + 2d = 5 + 2\left(\frac{3}{2}\right) = 5 + 3 = 8$

(iv) Let –4, □, □, □, □, 6 are the terms of A.P. $a, a + d, a + 2d, a + 3d, a + 4d, a + 5d$ respectively.

$a = -4$

and $\quad a + 5d = 6$

$-4 + 5d = 6$

$5d = 10$

$d = \frac{10}{5} = 2$

Hence, missing term

$a + d = -4 + 2 = -2$

$a + 2d = -4 + 4 = 0$

$a + 3d = -4 + 6 = 2$

$a + 4d = -4 + 8 = 4$

(v) Let □, 38, □, □, □, – 22 are the term of A.P. $a, a + d, a + 2d, a + 3d, a + 4d$ and $a + 5d$ respectively.

$a + d = 38$ ...(i)

$a + 5d = -22$ ...(ii)

Subtracting eqn. (i) from eqn. (ii), we get

$$\begin{aligned} a + 5d &= -22 \\ a + d &= 38 \\ - \quad - \quad &- \\ \hline 4d &= -60 \\ d &= \frac{-60}{4} = -15 \end{aligned}$$

Put the value of d in eqn. (i),

$a + (-15) = 38$

$a = 38 + 15 = 53$

Hence, missing term $a = 53$

$a + 2d = 53 + 2(-15) = 53 - 30 = 23$

$a + 3d = 53 + 3(-15) = 53 - 45 = 8$

$a + 4d = 53 + 4(-15) = 53 - 60 = -7$

4. Which term of the A. P. 3, 8, 13, 18, ... is 78 ?

Sol. : Let, nth term of A.P. 3, 8, 13, 18, ... is 78.

where, $a = 3, d = 8 - 3 = 5$

then $a_n = a + (n - 1)d$

$78 = 3 + (n - 1)5$

$78 - 3 = (n - 1)5$

$\frac{75}{5} = n - 1$

$15 = n - 1$

$n = 15 + 1 = 16$

Hence, the 16^{th} term of the given A.P. will be 78.

5. Find the number of terms in each of the following APs :

(i) 7, 13, 19, ... 205.

(ii) $18, 15\frac{1}{2}, 13, \ldots -47$.

Sol. : (i) 7, 13, 19, ... 205

Here, $a = 7, \quad d = 13 - 7 = 6$

and $a_n = 205$

then nth term $= a + (n - 1)d$

$205 = 7 + (n - 1)6$

$205 - 7 = (n - 1)6$

$198 = (n - 1)6$

$(n - 1) = \frac{198}{6} = 33$

$n = 33 + 1 = 34$

Hence, number of terms = 34

(ii) Given : $18, 15\frac{1}{2}, 13, \ldots -47$

$a = 18$, $d = 15\frac{1}{2} - 18 = \frac{31-36}{2} = \frac{-5}{2}$ and $a_n = -47$

then $a_n = a + (n-1)d$

$-47 = 18 + (n-1)\left(-\frac{5}{2}\right)$

$-47 - 18 = (n-1)\left(-\frac{5}{2}\right)$

$-65 \times \frac{2}{-5} = (n-1)$

$26 = n - 1$

$n = 26 + 1 = 27$

Hence, the total terms of A.P. = 27

6. **Check whether – 150 is a term of the A. P. 11, 8, 5, 2,........**

Sol. : Given AP :

11, 8, 5, 2

Here, $a = 11$, $d = 8 - 11 = -3$

Let, nth term of A.P. is – 150.

By, $a_n = a + (n-1)d$

$-150 = 11 + (n-1)(-3)$

$-150 - 11 = (n-1)(-3)$

$\frac{-161}{-3} = n - 1$

$n = 1 + \frac{161}{3} = \frac{164}{3}$

But n must be a positive integer. So our assumption is wrong. Hence, –150 is not a term of the given A.P.

7. **Find the 31st term of an A. P. whose 11th term is 38 and the 16th term is 73.**

Sol. : Let, First term of A.P. is a and Common difference is d

then $a_{11} = a + (11-1)d = 38$ [$\because a_{11} = 38$ Given,

$a + 10d = 38$ ...(i)

and $a_{16} = a + (16-1)d = 73$

$a + 15d = 73$ ...(ii)

Subtracting eqn. (ii) from eqn. (i), we get

$a + 10d = 38$

$a + 15d = 73$

$-\quad -\quad\quad -$

$-5d = -35$

$d = \frac{-35}{-5} = 7$

Put the value of d in eqn. (i),

$a + 10(7) = 38$

$a + 70 = 38$

$a = 38 - 70 = -32$

$\therefore$ 31^{th} term of A.P.

$= a + (31 - 1)d$

$= -32 + 30(7)$

$= -32 + 210 = 178$

8. An A. P. consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29^{th} term.

Sol. : Let, First term of A.P. is a and Common difference is d.

then according to question,

$a_3 = 12$

$a + 2d = 12$ ...(i)

and $a_{50} = 106$

$a + (50 - 1)d = 106$

$a + 49d = 106$ ...(ii)

Subtracting eqn. (ii) from eqn. (i), we get

$a + 2d = 12$

$a + 49d = 106$

$- \quad - \quad -$

$-47d = -94$

$d = \frac{-94}{-47} = 2$

Put the value of d in eqn. (i),

$a + 2(2) = 12$

$a = 12 - 4 = 8$

29^{th} term of A.P.

$a_{29} = a + 28d$

$= 8 + 28(2)$

$= 8 + 56$

$a_{29} = 64$

9. If the 3rd and the 9^{th} terms of an A. P. are 4 and – 8 respectively, which term of this A. P. is zero ?

Sol. : Let, First term of A.P. is a and Common difference is d

According to question,

$a + 2d = 4$ ...(i)

$a + 8d = -8$ ...(ii)

Subtracting eqn. (ii) from eqn. (i), we get

$a + 2d = 4$

$a + 8d = -8$

$- \quad - \quad +$

$-6d = 12$

$d = \frac{12}{-6} = -2$

Put the value of d in eqn. (i),

$a + 2(-2) = 4$

$a = 4 + 4 = 8$

Let, nth term is zero

$a_n = a + (n-1)d$

$0 = 8 + (n-1)(-2)$

$-8 = (n-1)(-2)$

$\frac{-8}{-2} = n - 1$

$4 = n - 1$

$n = 4 + 1 = 5$

10. The 17^th term of an A. P. exceeds it 10^th term by 7. Find the common difference.

Sol. : Let, First term = a and Common difference = d

Given : $a_{17} = a_{10} + 7$

$a + 16d = a + 9d + 7$

$a + 16d - (a + 9d) = 7$

$16d - 9d = 7$

$7d = 7$

$d = \frac{7}{7} = 1$

Hence, the common difference of A.P. is 1.

11. Which term of the A. P. 3, 15, 27, 39, ... will be 132 more than is 54^th term ?

Sol. : Given A.P. :

3, 15, 27, 39, ...

Here, $a = 3$, $d = 15 - 3 = 12$

Let, nth term exceed the 54th term by 132, then,

$a_n = a_{54} + 132$

$a + (n-1)d = a + 53d + 132$

$3 + (n-1)12 = 3 + 53 \times 12 + 132$

$(n-1)12 = 3 + 636 + 132 - 3$

$(n-1) = \frac{771-3}{12} = \frac{768}{12} = 64$

$n - 1 = 64$

$n = 64 + 1 = 65$

Hence, the value of 65^th term of the A.P. is 132 more than that of 54^th term.

12. Two APs have the same common difference, the difference between their 100^th terms is 100, what is the difference between their 1000^th terms ?

Sol. : Let, Two arithmetic progressions are $a_1, a_2, a_3, \dots a_n$ and $b_1, b_2, b_3, \dots b_n$.

Given : The common difference of both the A.P. is equal.

nth term of first A.P. $= a_n = a_1 + (n-1)d$ ...(i)

nth term of second A.P. $= b_n = b_1 + (n-1)d$ ...(ii)

Subtracting eqn. (ii) from eqn. (i), we get

$a_n - b_n = a_1 + (n-1)d - b_1 - (n-1)d$

$a_n - b_n = a_1 - b_1$

Now, According to question,

$a_{100} - b_{100} = a_1 - b_1 = 100$

$\therefore \quad a_{1000} - b_{1000} = a_1 - b_1 = 100$

Hence, the difference between the 1000th terms of both the arithmetic progressions is 100.

13. How many three digit numbers are divisible by 7 ?

Sol. : We know that the first 3 digit numbers is 105 and the last is 994 which are divisible by 7.

So, 105, 112, 119, ... 994

$\therefore \quad d = 112 - 105 = 7$

Let, there are n terms in the A.P.

then $\quad n^{th}$ term = 994

$a + (n-1)d = 994$

$105 + (n-1)7 = 994$

$7(n-1) = 994 - 105 = 889$

$n - 1 = \frac{889}{7} = 127$

$n = 127 + 1 = 128$

Hence, total number of 3-digit numbers which are divisible by 7 = 128

14. How many multiples of 4 lie between 10 and 250 ?

Sol. : The first integers between 10 and 250 is 12 which is multiple of 4 and 248 will be the largest integer.

Required A.P. : 12, 16, 20, ..., 248

$\therefore \quad a = 12$

$l = 248$

$d = 4$

Let, the number of terms of the series be n.

then $\quad l = a + (n-1)d$

$248 = 12 + (n-1)4$

$248 - 12 = (n-1)4$

$\frac{236}{4} = n - 1$

$n - 1 = 59$

$n = 59 + 1 = 60$

So, there are total 60 multiples of 4 between 10 and 250.

15. For what value of n are the n^{th} terms of two APs : 63, 65, 67, ... and 3, 10, 17, ... equal ?

Sol. : First term of first series (a) = 63

Common difference (d) = 65 – 63 = 2

First term of second series (a) = 3

Common difference (d) = 10 – 3 = 7

$\therefore$ The n^{th} term of both the series are equal.

$\therefore 63 + (n-1) \times 2 = 3 + (n-1) \times 7$

$63 + 2n - 2 = 3 + 7n - 7$

$$61 + 4 = 7n - 2n$$
$$65 = 5n$$
$$n = \frac{65}{5} = 13$$

Hence, the 13^{th} term of both the given series are equal.

16. Determine the A.P. whose third term is 16 and 7^{th} term exceeds the 5^{th} term by 12.

Sol. : Let, the first term of A.P. = a and Common difference = d

Given : Third term of A.P. = 16

$$a + 2d = 16 \quad \text{...(i)}$$

and 7^{th} term = 5^{th} term + 12

$$a + 6d = a + 4d + 12$$
$$a + 6d - a - 4d = 12$$
$$2d = 12$$
$$d = \frac{12}{2} = 6$$

Put the value of d in eqn. (i),

$$a + 2(6) = 16$$
$$a + 12 = 16$$
$$a = 16 - 12 = 4$$

Hence, the terms of A.P.

$$a = 4$$
$$a + d = 4 + 6 = 10$$
$$a + 2d = 4 + 12 = 16$$
$$a + 3d = 4 + 18 = 22$$

....

....

Hence, the A.P. is 4, 10, 16, 22, ...

17. Find the 20^{th} term from the last term of the A. P. 3, 8, 13, ... 253.

Sol. : Given : 3, 8, 13, ... 253 is an A.P.

Here, $a = 3$,

and $d = 8 - 3 = 5$

Last term (l) = 253

$\therefore$ 20^{th} term from last $= l - (n - 1)d$

$$= 253 - (20 - 1)5$$
$$= 253 - 19 \times 5$$
$$= 253 - 95 = 158$$

18. The sum of the 4^{th} and 8^{th} terms of an A. P. is 24 and the sum of the 6^{th} and 10^{th} terms is 44. Find the first three terms of the A. P.

Sol. : Let First term of A.P. is a and common difference is d then

$$a_4 + a_8 = 24$$
$$a + 3d + a + 7d = 24$$
$$2a + 10d = 24$$
$$a + 5d = 12 \quad \text{...(i)}$$

and $a_6 + a_{10} = 44$

$$a + 5d + a + 9d = 44$$

$$2a + 14d = 44$$

$$a + 7d = 22 \quad ...(ii)$$

Subtracting eqn. (ii) from eqn. (i),

$$a + 5d = 12$$

$$a + 7d = 22$$

$$- \quad - \quad -$$

$$2d = 10$$

$$d = \frac{10}{2} = 5$$

Put the value of d in eqn. (i),

$$a + 5(5) = 12$$

$$a = 12 - 25 = -13$$

Hence, first three terms of A.P.

$$a = -13$$

$$a + d = -13 + 5 = -8$$

$$a + 2d = -13 + 2(5) = -13 + 10 = -3$$

19. Subba Rao started work in 1995 at an annual salary of ₹ 5000 and received an increment of ₹ 200 each year. In which year did his income reach ₹ 7,000 ?

Sol. : The salaries received by Subba Rao in the years 1995, 1996, 1997, ... etc. are ₹ 5,000, ₹ 5,200, ₹ 5,400, ... ₹ 7,000 Hence, 5,000, 5,200, 5,400, ... 7,000 is an A.P.

Here, $a = 5000$

$$d = 5{,}200 - 5{,}000 = 200$$

Let, the total number of terms in the series is n, then

$$a_n = 7{,}000$$

$$a + (n - 1)d = 7{,}000$$

$$5000 + (n - 1)200 = 7{,}000$$

$$(n - 1)200 = 7{,}000 - 5{,}000$$

$$n - 1 = \frac{2{,}000}{200} = 10$$

$$n - 1 = 10$$

$$n = 10 + 1 = 11$$

Hence, in 11^{th} year his income will be reached ₹ 7,000.

20. Ramkali saved ₹ 5 in the first week of a year and then increased her weekly savings by ₹ 1.75. If in the n^{th} week, her weekly saving become ₹ 20.75, find n.

Sol. : Ramkali's weekly savings respectively

₹5, ₹(5 + 1.75), ₹(5 + 2 × 1.75) ...

i.e., ₹ 5, ₹ 6.75, ₹ 8.5, ...

Let, Ramkali saves ₹ 20.75 in n^{th} week

then

$5 + (n - 1) \times 1.75 \quad = 20.75$

$(n - 1) \times 1.75 = 20.75 - 5$

$(n - 1) = \frac{15.75}{1.75} = 9$

$n - 1 = 9$

$n = 9 + 1 = 10$

Exercise – 5.3

1. Find the sum of the following APs :

(i) 2, 7, 12, ... to 10 terms.

(ii) – 37, – 33, – 29, ... to 12 terms.

(iii) 0.6, 1.7, 2.8, ... to 100 terms.

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$, ... to 11 terms.

Sol. : (i) The given A.P. is as follow :

2, 7, 12, ... upto 10 terms

Here, $a = 2$, $d = 7 - 2 = 5$, $n = 10$

then $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2 \times 2 + (10-1)5]$

$= 5(4 + 45)$

$S_{10} = 5 \times 49 = 245$

(ii) Given : – 37, – 33, – 29, ... upto 12 terms

Here, $a = -37$, $d = -33 - (-37) = 4$, $n = 12$

The sum of first n terms of an A.P.

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{12} = \frac{12}{2}[2(-37) + (12-1)(4)]$

$= 6(-74 + 44) = 6 \times (-30)$

$S_{12} = -180$

(iii) Given : 0.6, 1.7, 2.8, ... upto 100 terms

Here, $a = 0.6$, $d = 1.7 - 0.6 = 1.1$

$n = 100$

The sum of first n terms of an A.P.

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{100} = \frac{100}{2}[2(0.6) + (100-1)(1.1)]$

$= 50(1.2 + 99 \times 1.1)$

$= 50(1.2 + 108.9) = 50 \times 110.1$

$S_{100} = 5505$

(iv) Given : $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, ...$upto 11 terms

Here, $a = \frac{1}{15}$, $d = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}$

and $n = 11$

then, the sum of first n terms of an A.P.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{11} = \frac{11}{2}\left[2 \times \frac{1}{15} + (11-1)\frac{1}{60}\right]$$

$$= \frac{11}{2}\left(\frac{2}{15} + \frac{10}{60}\right) = \frac{11}{2}\left(\frac{2}{15} + \frac{1}{6}\right)$$

$$= \frac{11}{2}\left(\frac{4+5}{30}\right) = \frac{99}{60} = \frac{33}{20}$$

2. Find the sums given below :

(i) $7 + 10\frac{1}{2} + 14 + ... + 84.$

(ii) 34 + 32 + 30 + ... + 10.

(iii) – 5 + (– 8) + (– 11) + ... + (– 230)

Sol. : (i) Given : $7 + 10\frac{1}{2} + 14 + ... + 84$

Here, First term (a) $= 7$,

Common difference $d = 10\frac{1}{2} - 7 = \frac{21-14}{2} = \frac{7}{2}$

and Last term (l)= 84

$\therefore$ $a_n = a + (n-1)d$

$$84 = 7 + (n-1)\frac{7}{2}$$

$$84 = \frac{14 + 7n - 7}{2}$$

$$168 = 7n + 7$$

$$168 - 7 = 7n$$

$$n = \frac{161}{7} = 23$$

Now, sum of n terms,

$$S_n = \frac{n}{2}(a + l)$$

$\therefore$ $S_{23} = \frac{23}{2}(7 + 84) = \frac{23 \times 91}{2}$

$$= \frac{2093}{2} = 1046.5$$

(ii) Given : 34 + 32 + 30 + ... + 10

Here, $a = 34$, $d = 32 - 34 = -2$,

and Last term (l) = 10

$\therefore$ $a_n = a + (n-1)d$

$$10 = 34 + (n-1)(-2)$$

$$= 34 - 2n + 2$$

$$10 = 36 - 2n$$

$$2n = 26$$

$$n = \frac{26}{2} = 13$$

Now, sum of n terms,

$$S_n = \frac{n}{2}(a + l)$$

$$\therefore \quad S_{23} = \frac{13}{2}(34 + 10)$$

$$= \frac{13 \times 44}{2} = 13 \times 22$$

$$= 286$$

(iii) Given : $-5 + (-8) + (-11) + ... + (-230)$

First term $a = -5$

$$d = -8 - (-5) = -8 + 5 = -3$$

and Last term $= -230$

$$\therefore \quad a_n = a + (n - 1)d$$

$$-230 = -5 + (n - 1)(-3)$$

$$-230 + 5 = (n - 1)(-3)$$

$$\frac{-225}{-3} = n - 1$$

$$n - 1 = 75$$

$$n = 75 + 1 = 76$$

Now, sum of n terms,

$$S_n = \frac{n}{2}(a + l)$$

$$\therefore \quad S_{76} = \frac{76}{2}[-5 + (-230)]$$

$$= 38(-235)$$

$$= -8930$$

3. **In an A. P. :**

(i) Given $a = 5, d = 3, a_n = 50$, find n and S_n.
(ii) Given $a = 7, a_{13} = 35$, find d and S_{13}.
(iii) Given $a_{12} = 37, d = 3$, find a and S_{12}.
(iv) Given $a_3 = 15, S_9 = 125$, find d and a_{10}.
(v) Given $d = 5, S_9 = 75$, find a and a_9.
(vi) Given $a = 2, d = 8, S_n = 90$, find n and a_n.
(vii) Given $a = 8, a_n = 62, S_n = 210$, find n and d.
(viii) Given $a_n = 4, d = 2, S_n = -14$, find n_a and a.
(ix) Given, $a = 3, n = 8, S = 192$, find d.
(x) Given, $l = 28, S = 144$, and there are total 9 terms. Find a.

Sol. : (i) Given : $a = 5, d = 3, a_n = 50$

$$\therefore \quad a_n = a + (n - 1)d$$

$$50 = 5 + (n - 1)(3)$$

$$50 - 5 = (n - 1)3$$

$$45 = (n - 1)3$$

$$\frac{45}{3} = n - 1$$

$n - 1 = 15$

$n = 15 + 1 = 16$

Now, sum of n terms,

$S_n = \frac{n}{2}(a + a_n)$

$= \frac{16}{2}(5 + 50)$

$= 8 \times 55 = 440$

Hence, $n = 16$ and $S_n = 440$

(ii) Given : $a = 7$, $a_{13} = 35$

$\therefore a + (13 - 1)d = 35$ $[\because a_n = a + (n - 1)d]$

$7 + 12d = 35$

$12d = 35 - 7$

$d = \frac{28}{12} = \frac{7}{3}$

Now, $S_n = \frac{n}{2}(a + a_n)$

$\therefore$ $S_{13} = \frac{13}{2}(a + a_{13})$

$= \frac{13}{2}(7 + 35) = \frac{13 \times 42}{2}$

$= 13 \times 21 = 273$

Hence, $d = \frac{7}{3}$

$S_n = 273$

(iii) Given : $a_{12} = 37$, $d = 3$

then, $a_{12} = 37$

$a + (12 - 1)d = 37$

$a + 11d = 37$

$a + 11 \times 3 = 37$

$a = 37 - 33 = 4$

Now, $S_{12} = \frac{12}{2}(a + a_{12})$

$= \frac{12}{2}(4 + 37)$

$= 6(41) = 246$

Hence, $a = 4$ and $S_{12} = 246$

(iv) Given : $a_3 = 15$

$a + 2d = 15$ ...(i)

$[\because a_n = a + (n - 1)d]$

and $S_{10} = 125$

$\frac{10}{2}[2a + (10 - 1)d] = 125$ $\left[\because S_n = \frac{n}{2}[2a + (n - 1)d]\right]$

$2a + 9d = \frac{125}{5}$

$2a + 9d = 25$ ...(ii)

Multiply eqn. (i) by 2 and subtracting it from eqn. (ii), we get

$$\begin{aligned} 2a + 9d &= 25 \\ 2a + 4d &= 30 \\ - \quad - \quad & - \\ \hline 5d &= -5 \\ d &= \frac{-5}{5} = -1 \end{aligned}$$

Put the value of d in eqn. (i),

$$a + 2(-1) = 15$$

$$a = 15 + 2 = 17$$

$$\therefore \quad a_{10} = a + (10 - 1)d$$

$$= 17 + 9(-1) = 17 - 9$$

$$= 8$$

Hence, $d = -1$ and $a_{10} = 8$

(v) Given : $d = 5$ and $S_9 = 75$

Let, First term a and Common difference $d = 5$, then

$$S_9 = 75$$

$$\frac{9}{2}[2a + (9 - 1)d] = 75 \qquad \left[\because S_n = \frac{n}{2}[2a + (n - 1)d]\right]$$

$$\frac{9}{2}(2a + 8d) = 75$$

$$9(a + 4d) = 75$$

$$a + 4d = \frac{75}{9} = \frac{25}{3}$$

Put $d = 5$,

$$a + 4 \times 5 = \frac{25}{3}$$

$$a = \frac{25}{3} - 20$$

$$= \frac{25 - 60}{3} = \frac{-35}{3}$$

and $\quad a_9 = a + (9 - 1)d \qquad [\because a_n = a + (n - 1)d]$

$$= \frac{-35}{3} + 8 \times 5$$

$$= \frac{-35}{3} + 40 = \frac{-35 + 120}{3}$$

$$= \frac{85}{3}$$

Hence, $a = \frac{-35}{3}$ and $a_9 = \frac{85}{3}$

(vi) Given : $a = 2$, $d = 8$ and $S_n = 90$

Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$$\frac{n}{2}[2a + (n - 1)d] = 90$$

$$\frac{n}{2}[2 \times 2 + (n - 1)8] = 90$$

$$n[2 + 4(n - 1)] = 90$$

$2n + 4n^2 - 4n = 90$

$4n^2 - 2n - 90 = 0$

$2n^2 - n - 45 = 0$

$2n^2 - 10n + 9n - 45 = 0$

$2n(n - 5) + 9(n - 5) = 0$

$(2n + 9)(n - 5) = 0$

$2n + 9 = 0$ or $n - 5 = 0$

$n = -\frac{9}{2}$ or $n = 5$

$n \neq -\frac{9}{2}$ because number of terms cannot be negative

$\therefore$ $n = 5$

Now, $a_n = a + (n - 1)d$

$a_5 = a + (5 - 1)d$

$= 2 + (5 - 1)8 = 2 + 4 \times 8$

$= 2 + 32 = 34$

Hence, $n = 5$ and $a_5 = 34$

(vii) Given : $a = 8$, $a_n = 62$, $S_n = 210$

$\therefore$ $a_n = 62$

$a + (n - 1)d = 62$ ...(i)

and $S_n = 210$

$\frac{n}{2}[2a + (n - 1)d] = 210$

$\frac{n}{2}[a + a + (n - 1)d] = 210$

$\frac{n}{2}(8 + 62) = 210$ [From eqn. (i)]

$\frac{n}{2} \times 70 = 210$

$n = \frac{210}{35} = 6$

From eqn. (i),

$a + (6 - 1)d = 62$

$8 + 5d = 62$ $[\because a = 8]$

$5d = 62 - 8$

$= 54$

$d = \frac{54}{5}$

Hence, $n = 6$ and $d = \frac{54}{5}$.

(viii) Let, First term be a then Given,

$a_n = 4$

$a + (n - 1)d = 4$

$a + (n - 1)2 = 4 \Rightarrow a + 2n - 2 = 4$

$\Rightarrow$ $a + 2n = 4 + 2 = 6$ ...(i)

and $S_n = \frac{n}{2}(a + a_n)$

$-14 = \frac{n}{2}(a+4)$

$-28 = n(a + 4)$ ...(ii)

From eqn. (i) $a = 6 - 2n$

Put the value of a in eqn. (ii),

$-28 = n(6 - 2n + 4)$

$-28 = n(10 - 2n)$

$-28 = 10n - 2n^2$

$2n^2 - 10n - 28 = 0$

$n^2 - 5n - 14 = 0$

$n^2 - 7n + 2n - 14 = 0$

$n(n - 7) + 2(n - 7) = 0$

$(n - 7)(n + 2) = 0$

$n - 7 = 0$ or $n + 2 = 0$

$n = 7$ or $n = -2$

$\therefore n = 7$

[$n \neq -2$ because number of terms cannot be negative]

Put the value of n in eqn. (i),

$a + 2(7) = 6$

$a = 6 - 14 = -8$

Hence, $n = 7$ and $a = -8$

(ix) Given : $a = 3$, $n = 8$,

$S = 192$

$\frac{n}{2}[2a+(n-1)d] = 192$

$\frac{8}{2}[2\times 3+(8-1)d] = 192$

$4(6 + 7d) = 192$

$6 + 7d = \frac{192}{4} = 48$

$7d = 48 - 6 = 42$

$d = \frac{42}{7} = 6$

(x) Let, the First term is a then

Given : $l = 28$, $n = 9$

and $S = 144$

$\frac{n}{2}(a+l) = 144$

$\frac{9}{2}(a+28) = 144$

$a + 28 = \frac{144\times 2}{9}$

$a + 28 = 32$

$a = 32 - 28 = 4$

4. **How many terms of the A. P. 9, 17, 25,.......must be taken to give a sum of 636?**

Sol. : Given : 9, 17, 25, ... is an A.P.

Here, $a = 9$ and $d = 17 - 9 = 8$

$\therefore \quad S_n = 636$

$\frac{n}{2}[2a + (n-1)d] = 636$

$\frac{n}{2}[2 \times 9 + (n-1)8] = 636$

$n(9 + 4n - 4) = 636$

$n(5 + 4n) = 636$

$4n^2 + 5n - 636 = 0$

$$n = \frac{-5 \pm \sqrt{5^2 - 4 \times 4 \times (-636)}}{2 \times 4}$$

$$= \frac{-5 \pm \sqrt{25 + 10176}}{8}$$

$$n = \frac{-5 \pm \sqrt{10201}}{8} = \frac{-5 \pm 101}{8}$$

$$n = \frac{-5 - 101}{8}$$

or $\quad n = \frac{-5 + 101}{8}$

or $\quad n = \frac{-106}{8} = \frac{-53}{4}$

or $\quad n = \frac{96}{8} = 12$

$\therefore \quad n = 12 \qquad$ [$\because n \neq \frac{-53}{4}$ number of terms cannot be negative]

5. **The first term of an A. P. is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.**

Sol. : Given : First term, $a = 5$

Last term $l = 45$, and sum = 400

Let, number of term is n and Common difference is d then

$S_n = 400$

$\Rightarrow \quad \frac{n}{2}(a + l) = 400$

$\Rightarrow \quad \frac{n}{2}(5 + 45) = 400$

$\Rightarrow \quad \frac{n}{2} \times 50 = 400$

$\Rightarrow \quad n = \frac{400}{25} = 16$

and $\quad l = 45$

$a + (n-1)d = 45$

$5 + (16-1)d = 45$

$15d = 45 - 5$

$d = \frac{40}{15} = \frac{8}{3}$

Hence, Common difference is $\frac{8}{3}$ and number of terms is 16.

6. The first and the last terms of an A. P. are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum ?

Sol. : Given : The First term of A.P., $a = 17$

Last term $(l) = 350$

Common difference $(d) = 9$

Let, number of terms is n, then

$$l = 350$$
$$a + (n-1)d = 350$$
$$17 + (n-1)9 = 350$$
$$(n-1)9 = 350 - 17$$
$$n - 1 = \frac{333}{9}$$
$$n - 1 = 37$$
$$n = 37 + 1 = 38$$

Now, $S_n = \frac{n}{2}(a + l)$

$$= \frac{38}{2}(17 + 350)$$
$$= 19(367) = 6973$$

Hence, number of terms is 38 and sum is 6973.

7. Find the sum of first 22 terms of an A.P. in which $d = 7$ and 22nd term is 149.

Sol. : Given :

$$d = 7 \text{ and } a_{22} = 149$$

then $a_{22} = 149$

$$a + 21d = 149 \quad [\because a_n = a + (n-1)d]$$
$$a + 21 \times 7 = 149$$
$$a = 149 - 147 = 2$$

Now, $S_{22} = \frac{n}{2}(a + a_{22})$

$$= \frac{22}{2}(2 + 149) = 11 \times 151 = 1661$$

Hence, the sum of the first 22 terms of the A.P. is 1661.

8. Find the sum of first 51 terms of an A. P. whose second and third terms are 14 and 18 respectively.

Sol. : Given : $a_2 = 14$, $a_3 = 18$

i.e., $a + d = 14$ ...(i)

and $a + 2d = 18$ ...(ii)

From eqn. (i) and (ii),

$$d = 18 - 14 = 4$$

Put the value of d in eqn. (i),

$$a + 4 = 14 \Rightarrow a = 14 - 4 = 10$$

Now, $S_{51} = \frac{51}{2}[2a + (51-1)d]$

$$\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$$

$$= \frac{51}{2}(2 \times 10 + 50 \times 4)$$

$= \frac{51}{2}(20 + 200) = \frac{51}{2}(220)$

$= 51 \times 110 = 5610$

Hence, the sum of 51 terms of the A.P. is 5610.

9. If the sum of first 7 terms of an A. P. is 49 and that of 17 terms is 289, find the sum of first *n* terms.

Sol. : Given : $S_7 = 49$, $S_{17} = 289$

$\therefore \quad S_7 = 49$

$\frac{7}{2}[2a + (7-1)d] = 49 \qquad \left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$\frac{7}{2}(2a + 6d) = 49$

$7(a + 3d) = 49$

$a + 3d = 7 \qquad ...(i)$

and $\quad S_{17} = 289$

$\frac{17}{2}[2a + (17-1)d] = 289$

$\frac{17}{2}(2a + 16d) = 289$

$17(a + 8d) = 289$

$a + 8d = \frac{289}{17}$

$a + 8d = 17 \qquad ...(ii)$

Subtracting eqn. (i) from eqn. (ii), we get

$a + 8d = 17$

$a + 3d = 7$

$- \quad - \qquad -$

$5d = 10$

$d = \frac{10}{5} = 2$

Put the value of d in eqn. (i), we get

$a + 3(2) = 7$

$a = 7 - 6 = 1$

Hence, $a = 1$ and $d = 2$

then $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2 \times 1 + (n-1)2]$

$= \frac{n}{2}(2 + 2n - 2)$

$S_n = \frac{n}{2}(2n) = n^2$

10. Show that $a_1, a_2, ..., a_n, ...$ form an A. P. where a_n is defined as below : Also find the sum of the first 15 terms in each case.

(i) $a_n = 3 + 4n$.

(ii) $a_n = 9 - 5n$.

Sol. : (i) Here, $a_n = 3 + 4n$,

Putting $n = 1, 2, 3, ...$

$a_1 = 3 + 4(1) = 7$

$a_2 = 3 + 4(2) = 11$

$a_3 = 3 + 4(3) = 15$

$a_4 = 3 + 4(4) = 19$

....

....

$a_{n-1} = 3 + 4(n - 1) = 3 + 4n - 4 = 4n - 1$

$a_n = 3 + 4n$

....

....

Now, $a_2 - a_1 = 11 - 7 = 4$

$a_3 - a_2 = 15 - 1 = 4$

$a_4 - a_3 = 19 - 15 = 4$

....

....

$a_n - a_{n-1} = (3 + 4n) - (4n - 1) = 4$

....

....

Here, the common difference (d) = 4 is equal in each case.

Hence, the obtained series is A.P.

Here, $a = 7$, $d = 4$ and $n = 15$

then $S_n = \frac{n}{2}[2a + (n - 1)d]$

$$S_{15} = \frac{15}{2}[2(7) + (15 - 1)4]$$

$$= \frac{15}{2}[14 + 14 \times 4] = \frac{15}{2}(14 + 56)$$

$$= \frac{15}{2}(70) = 15 \times 35 = 525$$

(ii) Here : $a_n = 9 - 5n$.

Putting $n = 1, 2, 3, ...$

$a_1 = 9 - 5(1) = 4$

$a_2 = 9 - 5(2) = -1$

$a_3 = 9 - 5(3) = -6$

$a_4 = 9 - 5(4) = -11$

....

....

$a_{n-1} = 9 - 5(n - 1) = 9 - 5n + 5 = 14 - 5n$

$a_n = 9 - 5n$

....

....

Now, $a_2 - a_1 = -1 - 4 \quad = -5$

$a_3 - a_2 = -6 - (-1) = -5$

$a_4 - a_3 = -11 - (-6) \quad = -5$

....

....

$a_n - a_{n-1} = (9 - 5n) - (14 - 5n) = -5$

....

....

Here, common difference $(d) = -5$ is equal in each case.

Hence, the obtained series is A.P.

Now, Here $a = 4$, $d = -5$, $n = 15$

then $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{15} = \frac{15}{2}[2(4) + (15-1)(-5)]$$

$$= \frac{15}{2}[8 + 14(-5)] = \frac{15}{2}(8 - 70)$$

$$= \frac{15}{2} \times (-62)$$

$$= -465$$

11. If the sum of the first n terms of an A. P. is $4n - n^2$, what is the first term (that is S_1) ? What is the sum of first two terms ? What is the second term ? Similarly, find the 3^{rd}, the 10^{th} and the n^{th} terms.

Sol. : Given : $S_n = 4n - n^2$

Put $n = 1$, $\quad S_1 = 4(1) - 1^2 = 3$ (First term)

Put $n = 2$, $\quad S_2 = 4(2) - 2^2 = 4$

$\therefore$ n^{th} term of A.P.

$$a_n = S_n - S_{n-1}$$

$\therefore$ Second term $\quad a_2 = S_2 - S_1$

$= 4 - 3 = 1$

$\therefore$ Third term $\quad a_3 = S_3 - S_2$

$= [4(3) - 3^2] - 4$

$= -1$

$\therefore$ Tenth term $= S_{10} - S_9$

$= [4(10) - 10^2] - [4(9) - 9^2]$

$= -60 - (-45) = -60 + 45$

$= -15$

$\therefore$ nth term $a_n = S_n - S_{n-1}$

$= (4n - n^2) - [4(n-1) - (n-1)^2]$

$= 4n - n^2 - [4n - 4 - n^2 - 1 + 2n]$

$= 4n - n^2 - 6n + n^2 + 5$

$$= -2n + 5$$

or $a_n = 5 - 2n$

12. Find the sum of the first 40 positive integers divisible by 6.

Sol. : The positive integers divisible by 6 are 6, 12, 18, ... respectively.

Here, First term $a = 6$,

Common difference $d = 12 - 6 = 6$

Number of terms $= 40$

then $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{40} = \frac{40}{2}[2 \times 6 + (40-1)6]$$

$$= 20(12 + 39 \times 6)$$

$$= 20(12 + 234) = 20 \times 246$$

$$= 4920$$

13. Find the sum of the first 15 multiples of 8.

Sol. : First multiple of 8, $8 \times 1, 8 \times 2, 8 \times 3, \ldots 8 \times 15$

i.e., 8, 16, 24, ... 120.

Here, First term $a = 8$, Common difference $d = 16 - 8 = 8$ Number of terms $= 15$ and Last term $l = 120$

then, $S_n = \frac{n}{2}(a + l)$

$$S_{15} = \frac{15}{2}(8 + 120) = \frac{15 \times 128}{2}$$

$$= 960$$

14. Find the sum of the odd numbers between 0 and 50.

Sol. : The odd numbers between 0 and 50 are 1, 3, 5, 7,.....,49

Here, First term $a = 1$, $d = 3 - 1 = 2$ and Last term $l = 49$

Let, number of terms be n then

$$l = a + (n-1)d$$

$$49 = 1 + (n-1)2$$

$$48 = (n-1)2$$

$$\frac{48}{2} = n - 1$$

$$n = 24 + 1 = 25$$

Now, $S_n = \frac{n}{2}(a + l)$

$$S_{25} = \frac{25}{2}(1 + 49) = 25 \times 25 = 625$$

15. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows : ₹ 200 for the first day, ₹ 250 for the second day, ₹ 300 for the third day, etc., the penalty for each succeeding day being ₹ 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days ?

Sol. : Here, the penalty for each succeeding day is ₹ 50 more than the previous day, the penalty of first, second, third days etc. are in A.P.

i.e., ₹ 200, ₹ 250, ₹ 300, ... are in A.P.

Here,, $a = 200$,

$d = 250 – 200 = 50$

If the contractor completes the work 30 days late, then he will have to pay S_{30} fine as penalty *i.e.*,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore \quad S_{30} = \frac{30}{2}[2 \times 200 + (30-1)50]$$

$= 15(400 + 29 \times 50)$

$= 15(400 + 1450)$

$= 15(1850) = 27,750$

Hence, the late penalty of 30 days is ₹ 27,750.

16. A sum of ₹ 700 is to be used to given seven cash prizes to students of a school for their overall academic performance. If each prize is ₹ 20 less than its preceding prize, find the value of each of the prizes.

Sol. : Let, amount of first prize = ₹ a

and each prize is ₹ 20 less than the previous prize, then the order of the 7 prizes received is as follows :

₹ a, ₹ $(a – 20)$, ₹ $(a – 40)$, ... ₹ $(a – 120)$

Here, First term = a

Common difference (d)= – 20

Last term $(l) = a – 120$

Number of terms $n = 7$

Total amount $S_7 = 700$

$$\frac{7}{2}[a + l] = 700$$

$$\frac{7}{2}[a + a - 120] = 700$$

$2a – 120 = 200$

$2a = 320$

$a = 160$

Hence, the amount of each prize is ₹ 160, ₹ 140, ₹ 120,₹ 100, ₹ 80, ₹ 60 and ₹ 40.

17. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, *e.g.*, a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students ?

Sol. : According to question, there are three section of each class. So every class has planted plants. *i.e.*, the plants planted by Class I, Class II, Class III,.....Class XII are as follows :

$1 \times 3, 2 \times 3, 3 \times 3, ... 12 \times 3$

Clearly, 3, 6, 9, ..., 36 are in A.P.

Here, First term, $a = 3$, Common difference = 6 – 3 = 3 and Last term $(l) = 36$

Now, $l = a + (n – 1)d$

$36 = 3 + (n – 1)3$

$36 = 3 + 3n - 3$

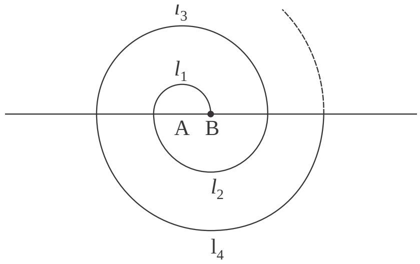
$36 = 3n$

$n = \frac{36}{3} = 12$

Hence, the number of plants planted by the students.

$S_{12} = \frac{12}{2}(3+36) = 6 \times 39 = 234$ $\left[\because S_n = \frac{n}{2}(a+l)\right]$

18. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ... as shown in Figure below. What is the total length of such a spiral made up of thirteen consecutive semicircles ? $\left[\text{Take } \pi = \frac{22}{7}\right]$



Sol. : A spiral is made up of 13 consecutive semicircles and the radii of 13 consecutive semicircles are 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm,.... .

So, the Length of first semicircle $l_1 = 0.5\,\pi$ cm [$\because$ Perimeter of semicircle = $\pi \times$ radius]

Length of second semicircle $l_2 = 1.0\pi = \pi$ cm

Length of third semicircle $l_3 = 1.5\pi$ cm

Length of fourth semicircle $l_4 = 2\pi$ cm

....

....

So, obtained series :

$0.5\pi, \pi, 1.5\pi, 2\pi$, ... which are in A.P.

Here, First term $a = 0.5\pi$

Common difference $d = \pi - 0.5\pi = 0.5\pi$

and number of terms = 13

$\therefore$ T the sum of n terms

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{13} = \frac{13}{2}[2 \times 0.5\pi + (13-1)0.5\pi]$$

$$= \pi \times 0.5 \times \frac{13}{2}(14)$$

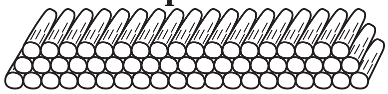
$$= \pi \times 0.5 \times 13 \times 7$$

$$= \frac{22}{7} \times \frac{5}{10} \times 13 \times 7$$

$$= 143 \text{ cm}$$

Hence, the total length of such a spiral made up of 13 consecutive semicircles is 143 cm.

19. 200 logs are stacked in the following manner : 20 logs in the bottom row, 19 logs in the next row, 18 logs in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row ?



Sol. : Stacked is placed as a series 20, 19, 18, 17 in each row. Clearly this is an A.P.

Here, $a = 20$, $d = 19 - 20 = -1$ and $S_n = 200$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$200 = \frac{n}{2}[2 \times 20 + (n-1)(-1)]$

$200 = \frac{n}{2}(40 - n + 1)$

$400 = n(41 - n)$

$400 = 41n - n^2$

$n^2 - 41n + 400 = 0$

$n^2 - 25n - 16n + 400 = 0$

$n(n - 25) - 16(n - 25) = 0$

$(n - 25)(n - 16) = 0$

$n = 25$ or $n = 16$

So, the number of rows is either 25 or 16

When $n = 16$, $a_n = a + (n-1)d$

$a_{16} = 20 + 15(-1)$

$= 20 - 15 = 5$

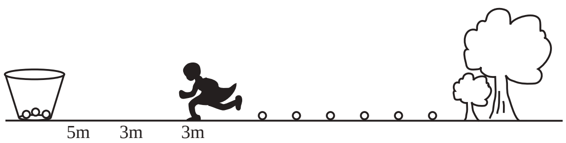
When $n = 25$ then, $a_n = a + (n-1)d$

$a_{25} = 20 + 24(-1)$

$= 20 - 24 = -4$

∴ The number of rows stacked cannot be negative. So, the number of rows is 16 and the number of stacked in the top most row is 5.

20. In a potato race, a bucket is placed at the starting point, which is 5m from the first potato, and the other potatoes are placed 3m apart in a straight line. There are ten potatoes in the line.



A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs hack to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run ?

Sol. : A competitor starts from the bucket, pickup the nearest potato, which is 5 m away, and puts it back in the bucket, then the distance travelled to put the first potato in the bucket = 2 × 5 = 10m

Similarly, picking up the second potato which is 3 m away from the first potato, comes back and drops it in the bucket, then

Distance taken to put the second potato in the bucket

$= 2 \times (5 + 3)$

$= 16$ m

Similarly, distance taken to put the third potato in the bucket $2 \times (5 + 3 + 3) = 22$m

So, obtained series

10, 16, 22, 28, ...

which is an A.P.

Here, First term $a = 10$, Common term $d = 16 - 10 = 6$

and number of terms = 10

$\therefore$ Sum of n terms $S_n = \frac{n}{2}[2a + (n - 1)d]$

$$S_{10} = \frac{10}{2}[2 \times 10 + (10 - 1)6]$$

$= 5(20 + 54) = 5 \times 74$

$= 370$

Exercise – 5.4

1. Which term of the A. P. 121, 117, 113, ... is its first negative term ?

Sol. : Given : 121, 117, 113, ... is a A.P.

First term $a = 121$ and Common difference $d = 117 - 121 = -4$

Since, there are n terms in A.P.

$\therefore \quad a_n = a + (n - 1)d$

$= 121 + (n - 1)(-4)$

$= 121 - 4n + 4 = 125 - 4n$

For the first negative term $a_n < 0$

$125 - 4n < 0$

$125 < 4n$

$4n > 125$

$n > \frac{125}{4}$

$n > 31\frac{1}{4}$

$\therefore$ Minimum value of n is 32.

Hence, 32^{nd} terms of A.P. will be first negative term.

2. The sum of the third and the seventh terms of an A.P. is 6 and their product is 8. Find the sum of first sixteen terms of the A.P.

Sol. : Let, first term of A.P. be a and common difference is d then,

Given : $a_3 + a_7 = 6$

$a + 2d + a + 6d = 6$ $\quad [\because a_n = a + (n - 1)d]$

$2a + 8d = 6$

$a + 4d = 3$ ...(i)

and $a_3 . a_7 = 8$

$(a + 2d)(a + 6d) = 8$

$\{(a + 4d) - 2d\}\{(a + 4d) + 2d\} = 8$

$\{3 - 2d\}\{3 + 2d\} = 8$ [From eqn. (i)]

$9 - 4d^2 = 8$

$4d^2 = 1$

$d^2 = \frac{1}{4}$

$d = \pm\frac{1}{2}$

$d = \frac{1}{2}$ Putting in eqn. (i),

$a + 4 \times \frac{1}{2} = 3 \Rightarrow a + 2 = 3$

$\Rightarrow a = 3 - 2 = 1$

$\therefore$ Sum of first 16 terms $= \frac{16}{2}[2a + (16 - 1)d]$

$= 8\left(2 \times 1 + 15 \times \frac{1}{2}\right)$

$= 8\left(\frac{4 + 15}{2}\right) = 8 \times \frac{19}{2}$

$= 4 \times 19 = 76$

$d = -\frac{1}{2}$ Putting in eqn. (i),

$a + 4 \times \left(-\frac{1}{2}\right) = 3$

$a + (-2) = 3$

$a = 3 + 2 = 5$

$\therefore$ Sum of first 16 terms $= \frac{16}{2}[2a + (16 - 1)d]$

$= 8\left[2 \times 5 + 15 \times \left(-\frac{1}{2}\right)\right]$

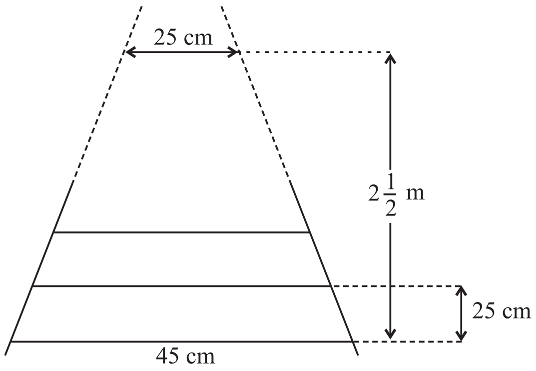
$= 8\left(10 - \frac{15}{2}\right)$

$= 8\left(\frac{20 - 15}{2}\right)$

$= 8 \times \frac{5}{2} = 20$

Hence, the sum of the first 16 terms of the A.P. will be 20 or 76.

3. **A ladder has rungs 25 cm apart. The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs ?**



Sol. : According to question, number of rungs

$$= \frac{2\frac{1}{2}\text{ m}}{25\text{ cm}} + 1$$

$$= \frac{250}{25} + 1 = 11 \qquad \left[\because 2\frac{1}{2}\text{m} = 250\text{ cm}\right]$$

Hence, the length of wood required to make the rungs

$$= \frac{11}{2}(25 + 45) \qquad \left[\because S_n = \frac{n}{2}(a + l)\right]$$

$$= \frac{11}{2} \times 70 = 385 \text{ cm}$$

4. **The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of *x* such that the sum of the numbers of the houses preceding the house numbered *x* is equal to the sum of the numbers of the houses following it. Find this value of *x*.**

Sol. : The houses in a row are consecutively numbered 1, 2, 3, ...49. It is clear that the list of numbers forms an A.P.

Here, $a = 1$, $d = 2 - 1 = 1$

$$\because \quad S_x = \frac{x}{2}[2a + (x-1)d]$$

$$= \frac{x}{2}[2 \times 1 + (x-1)1]$$

$$= \frac{x}{2}(2 + x - 1) = \frac{x}{2}(x + 1)$$

$$\therefore \quad S_{x-1} = \frac{x-1}{2}[2 \times 1 + (x - 1 - 1) \times 1]$$

$$= \frac{x-1}{2}(2 + x - 2) = \frac{x-1}{2}(x)$$

or $$S_{49} = \frac{49}{2}[2 \times 1 + (49 - 1) \times 1]$$

$$= \frac{49}{2}(2 + 48)$$

$$= \frac{49}{2} \times 50 = 1225$$

A.T.Q., $$S_{x-1} = S_{49} - S_x$$

$$\frac{x^2 - x}{2} = 1225 - \frac{x^2 + x}{2}$$

$$\frac{x^2 - x}{2} + \frac{x^2 + x}{2} = 1225$$

$$\frac{2x^2}{2} = 1225$$

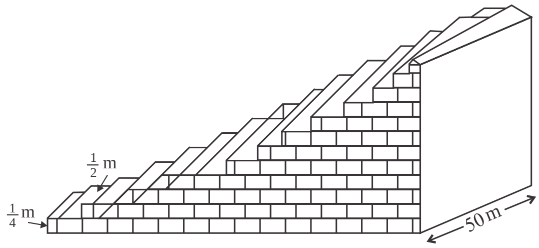
$$x^2 = 1225$$

$$x = \pm 35$$

$\because$ x cannot be negative.

$\therefore$ $x = 35$

5. **A small terrace at a football ground comprised of 15 steps each of which is 50 m long and built of solid concrete.**
Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m. Calculate the total volume of concrete required to build the terrace.



Sol. : Given, Length of each ladder = 50 m

Breadth of each ladder = $\frac{1}{2}$ m

Height of first ladder = $\frac{1}{4}$ m

Height of second ladder = $2 \times \frac{1}{4} = \frac{1}{2}$ m

Height of third ladder = $3 \times \frac{1}{4} = \frac{3}{4}$ m

That is, the height of each ladder is increased by $\frac{1}{4}$m and the length and breadth will remain the same.

So, volume of concrete used to make first ladder

$$= 50 \times \frac{1}{2} \times \frac{1}{4}$$

$$= \frac{25}{4} \text{ m}^3$$

Volume of concrete used to make second ladder

$$= 50 \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{50}{4} \text{ m}^3$$

Volume of concrete used to make third ladder

$$= 50 \times \frac{1}{2} \times \frac{3}{4}$$

$$= \frac{75}{4} \text{ m}^3$$

Obtained series : $\frac{25}{4}, \frac{50}{4}, \frac{75}{4}, ...$

Hence, the series is an A.P.

Here, $a = \frac{25}{4}$, $d = \frac{25}{4}$

Volume of concrete used to make platform of 15 ladder

$$S_{15} = \frac{15}{2}[2a + (15-1).d]$$

$$= \frac{15}{2}\left[2 \times \frac{25}{4} + 14 \times \frac{25}{4}\right]$$

$$= \frac{15}{2} \times 2 \times \frac{25}{4}[1+7]$$

$$= \frac{15}{2} \times 2 \times \frac{25}{4} \times 8$$

$$= 750 \text{ m}^3$$

Triangles 6

NCERT SOLUTIONS

What's inside

– In-Chapter Q's (solved)
– Textbook Exercise Q's (solved)

Exercise – 6.1

1. Fill in the blanks using the correct word given in brackets :

(i) All circles are .... (congruent, similar)

(ii) All squares are .... (similar, congruent)

(iii) All .... triangles are similar. (isosceles, equilateral)

(iv) Two polygons of the same number of sides are similar, if

(a) their corresponding angles are .... and

(b) their corresponding sides are .... (equal, proportional)

Sol. :(i) Similar because the shape of the circles are same, but the measures are not equal.

(ii) Similar because the shape of the squares are same, but the measures are not equal.

(iii) Equilateral, because the shape of all the equilateral triangles are same, but their measures are not equal. So, all equilateral triangles are similar.

(iv) (a) Equal, because if the corresponding angles are equal of two polygons of the same number of sides, then they are similar.

(b) Proportional, because if the corresponding sides are proportional of the same number of sides, then they are similar.

2. Give two different examples of pair of :

(i) Similar figures.

(ii) Non-similar figures.

Sol. :(i) Two figures which are similar :

(a) Pair of squares are similar.

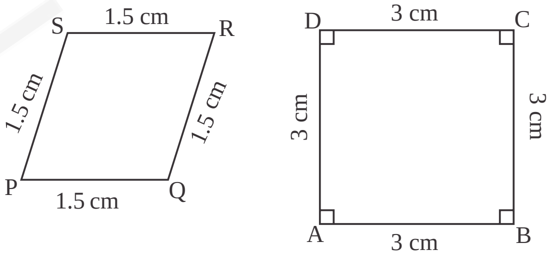
(b) Pair of equilateral triangles are similar

(ii) Two figures which are non-similar :

(a) A square and a rectangle are not similar

(b) A square and a circle are not similar.

3. State whether the following quadrilaterals are similar or not :



Sol. : Here, $\frac{PQ}{AB} = \frac{1.5}{3} = \frac{1}{2}$

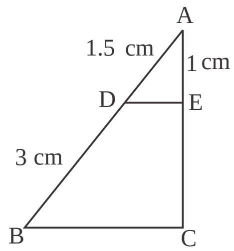
and $\frac{QR}{BC} = \frac{1.5}{3} = \frac{1}{2}$

The corresponding sides of both the quadrilateral *PQRS* and *ABCD* are proportional, but their corresponding angles are not equal.

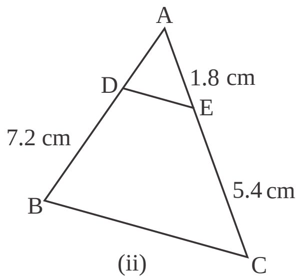
Hence, given quadrilateral are not similar.

Exercise – 6.2

1. In figure below, (i) and (ii), $DE \parallel BC$. Find EC in (i) and AD in (ii).



(i)



(ii)

Sol. : In figure (i), $DE \parallel BC$

$$\frac{AD}{BD} = \frac{AE}{EC}$$ [By basic proportionality theorem]

$$\Rightarrow \quad \frac{1.5}{3} = \frac{1}{EC}$$

$$\therefore \quad EC = \frac{3}{1.5} = 2 \text{ cm}$$

In figure (ii), $DE \parallel BC$

$$\frac{AD}{BD} = \frac{AE}{EC}$$ [By basic proportionality theorem]

$$\Rightarrow \quad \frac{AD}{7.2} = \frac{1.8}{5.4}$$

$$\Rightarrow \quad \frac{AD}{7.2} = \frac{1}{3}$$

$$\therefore \quad AD = \frac{7.2}{3} = 2.4 \text{ cm}$$

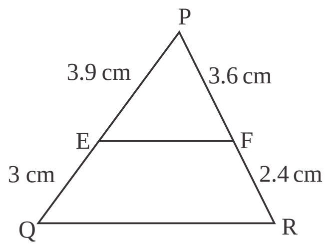
2. E and F are points on the sides PQ and PR respectively of a $\triangle PQR$. For each of the following cases, state whether $EF \parallel QR$:

(i) $PE = 3.9$ cm, $EQ = 3$ cm, $PF = 3.6$ cm and $FR = 2.4$ cm

(ii) $PE = 4$ cm, $QE = 4.5$ cm, $PF = 8$ cm and $RF = 9$ cm

(iii) $PQ = 1.28$ cm, $PR = 2.56$ cm, $PE = 0.18$ cm and $PF = 0.36$ cm

Sol. : **(i)**



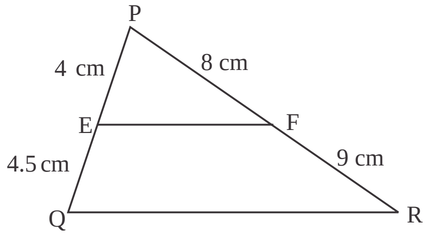
From figure, $\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$

and $\frac{PF}{FR} = \frac{3.6}{2.4} = \frac{3}{2} = 1.5$

$$\therefore \quad \frac{PE}{EQ} \neq \frac{PF}{FR}$$

Thus, EF is not parallel to QR, because here the basic proportionality theorem is not satisfied.

(ii)



From figure,

$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{40}{45} = \frac{8}{9}$$

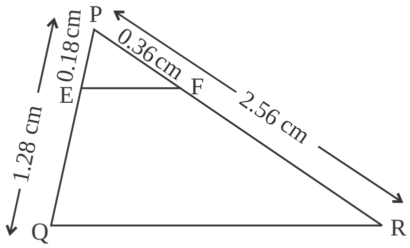
and $\quad \frac{PF}{FR} = \frac{8}{9}$

Here, $\quad \frac{PE}{EQ} = \frac{PF}{FR}$

∴ The basic proportionality theorem is satisfied.

∴ $\quad EF \parallel QR$

(iii)



From figure,

$$EQ = PQ - PE$$
$$= 1.28 - 0.18$$
$$= 1.1 \text{ cm}$$

and $\quad FR = 2.56 - 0.36$
$$= 2.2 \text{ cm}$$

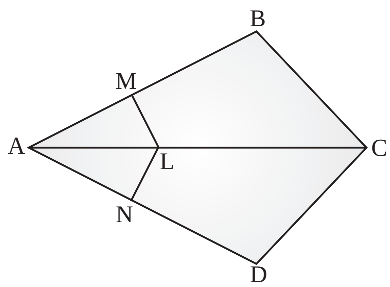
Hence, $\quad \frac{PE}{EQ} = \frac{0.18}{1.1} = \frac{18}{110} = \frac{9}{55}$

$$\frac{PF}{FR} = \frac{0.36}{2.2} = \frac{36}{220} = \frac{9}{55}$$

Here, $\quad \frac{PE}{EQ} = \frac{PF}{FR}$

Hence, $EF \parallel QR$, because here the basic proportionality theorem is satisfied.

3. In figure below, if $LM \parallel CB$ and $LN \parallel CD$, prove that $\frac{AM}{AB} = \frac{AN}{AD}$.



Sol. : In ΔACB, $LM \parallel CB$ [Given]

$$\frac{AM}{MB} = \frac{AL}{LC} \quad ...(i)$$

[By basic proportionality theorem]

In ΔACD, $LN \parallel CD$

$$\frac{AN}{ND} = \frac{AL}{LC} \quad ...(ii)$$

[By basic proportionality theorem]

By eqn. (i) and (ii),

$$\frac{AM}{MB} = \frac{AN}{ND}$$

or $$\frac{MB}{AM} = \frac{ND}{AN}$$

Adding 1 to both sides,

$$\frac{MB}{AM} + 1 = \frac{ND}{AN} + 1$$

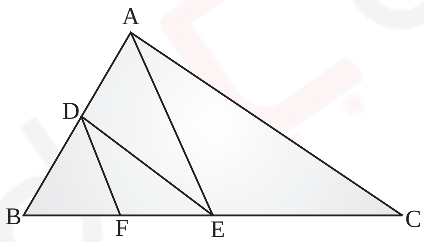
or $$\frac{MB + AM}{AM} = \frac{ND + AN}{AN}$$

or $$\frac{AB}{AM} = \frac{AD}{AN}$$

or $$\frac{AM}{AB} = \frac{AN}{AD}$$

Hence Proved

4. In figure below, *DE* || *AC* and *DF* || *AE*. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.



Sol. : In ΔBAC, $DE \parallel AC$

$$\therefore \quad \frac{BE}{EC} = \frac{BD}{DA} \quad ...(i)$$

[By basic proportionality theorem]

In ΔBAE, $DF \parallel AE$

$$\frac{BF}{FE} = \frac{BD}{DA} \quad ...(ii)$$

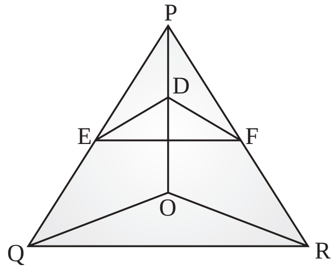
[By basic proportionality theorem]

By eqn. (i) and (ii),

$$\frac{BE}{EC} = \frac{BF}{FE} \text{ or } \frac{BF}{FE} = \frac{BE}{EC}$$

Hence Proved

5. In figure below, *DE* || *OQ* and *DF* || *OR*. Show that *EF* || *QR*.



Sol. : Given, $DE \parallel OQ$ and $DF \parallel OR$

In ΔPQO, $\frac{PE}{EQ} = \frac{PD}{DO}$ ...(i)

[By basic proportionality theorem]

In ΔPOR, $\frac{PF}{FR} = \frac{PD}{DO}$ ...(ii)

[By basic proportionality theorem]

By eqn. (i) and (ii),

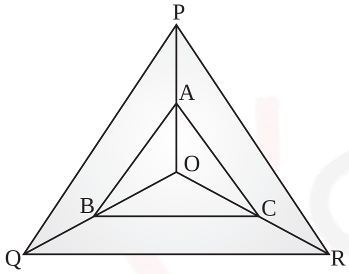
$$\frac{PE}{EQ} = \frac{PF}{FR}$$

Now, in ΔPQR, $\frac{PE}{EQ} = \frac{PF}{FR}$ [Have proved above]

$\therefore$ By the converse of the basic proportionality theorem

$EF \parallel QR$ **Hence Proved**

6. In figure below, *A*, *B* and *C* are points on *OP*, *OQ* and *OR* respectively such that *AB* || *PQ* and *AC* || *PR*. Show that *BC* || *QR*.



Sol. : Given, in ΔOPQ,

$AB \parallel PQ$

$\therefore$ $\frac{OA}{AP} = \frac{OB}{BQ}$ ...(i)

[By basic proportionality theorem]

In ΔOPR, $AC \parallel PR$

$\frac{OA}{AP} = \frac{OC}{CR}$ ...(ii)

[By basic proportionality theorem]

By eqn. (i) and (ii),

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

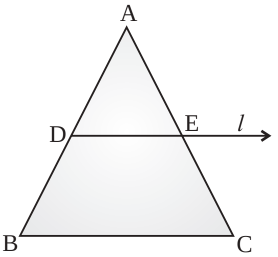
By the converse of the basic proportionality theorem,

$BC \parallel QR$ **Hence Proved**

7. Using theorem 1 (Basic Proportionality Theorem), Prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side.

Sol. : Let D be the mid-point of AB in ΔABC.

then, $\frac{AD}{DB} = 1$ ...(i)



Through the point D draw a line l which meets AC at point E and $l \parallel BC$.

By basic proportionality theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\therefore \quad \frac{AE}{EC} = 1 \qquad \text{[From eqn. (i)]}$$

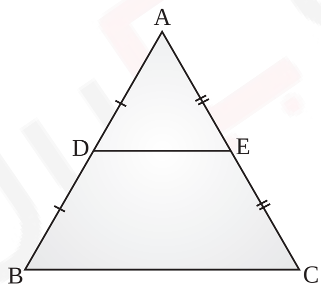
Therefore, E is the mid-point of AC. Hence, a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side.

8. Using theorem 2 (Converse of basic proportionality theorem), prove that the line joining the mid-point of any two sides of a triangles is parallel to the third side.

Sol. : Let D and E be the mid-point of AB and AC in the ΔABC

then $\frac{AD}{DB} = 1$ ...(i)

and $\frac{AE}{EC} = 1$ ...(ii)



By eqn. (i) and (ii),

$$\frac{AD}{DB} = \frac{AE}{EC}$$

By converse of the basic proportionality theorem,

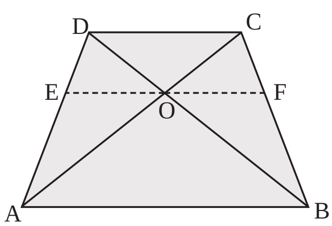
$$DE \parallel BC$$

Hence, the line joining the mid-point of any two sides of a triangle is parallel to the third side.

9. *ABCD* is trapezium in which *AB* || *DC* and its diagonals intersect each other at the point *O*. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Sol. : Given, $ABCD$ is a trapezium in which $AB \parallel DC$ and its diagonals AC and BD intersect at point O.

To Prove : $\frac{AO}{BO} = \frac{CO}{DO}$



Construction : A line segment EF is drawn through the point O such that $EF \parallel DC$.

Proof : In ΔACD, $OE \parallel CD$ [From construction]

$$\frac{AE}{ED} = \frac{AO}{OC} \quad ...(i)$$

[By basic proportionality theorem]

In $\Delta ABD, OE \parallel BA$

$$\frac{DE}{EA} = \frac{DO}{OB}$$ [By basic proportionality theorem]

$$\frac{AE}{ED} = \frac{OB}{OD} \quad ...(ii)$$

By eqn. (i) and (ii),

$$\frac{AO}{OC} = \frac{OB}{OD}$$

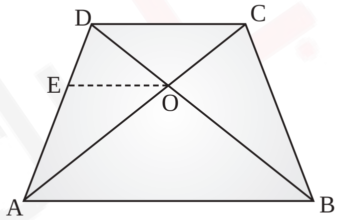
or $$\frac{AO}{BO} = \frac{OC}{OD}$$ **Hence Proved**

10. The diagonals of a quadrilateral $ABCD$ intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that $ABCD$ is a trapezium.

Sol. : Given, $ABCD$ is a quadrilateral whose diagonals AC and BD intersect at the point O such that,

$$\frac{AO}{BO} = \frac{CO}{DO} \text{ or } \frac{AO}{OC} = \frac{BO}{OD} \quad ...(i)$$

Draw $EO \parallel BA$ through the point O. Line OE intersects AD at point E.



In ΔDAB,

$$EO \parallel AB$$

$$\therefore \quad \frac{DE}{EA} = \frac{DO}{OB}$$

or $$\frac{AE}{ED} = \frac{BO}{OD} \quad ...(ii)$$

By eqn. (i) and (ii),

$$\frac{AO}{CO} = \frac{AE}{ED} \Rightarrow OE \parallel CD$$ [By converse of the basic proportionality theorem]

Now, $BA \parallel OE$ ...(iii)

and $OE \parallel CD$ ...(iv)

From eqn. (iii) and (iv),

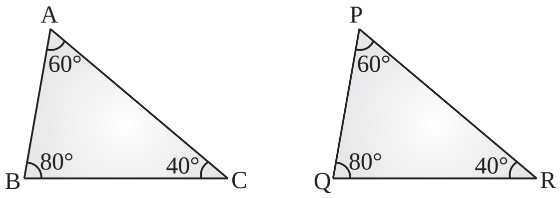
$$AB \parallel CD$$

Hence, quadrilateral $ABCD$ is a trapezium.

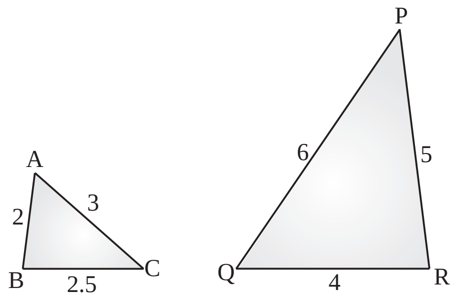
Exercise – 6.3

1. State which pairs of triangles in figure below are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form :

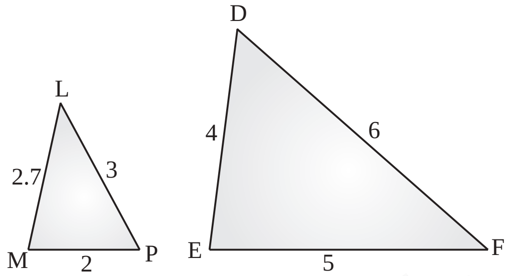
(i)



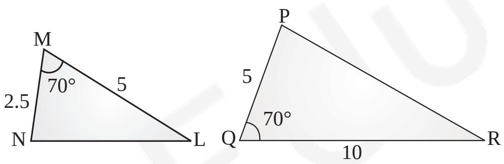
(ii)



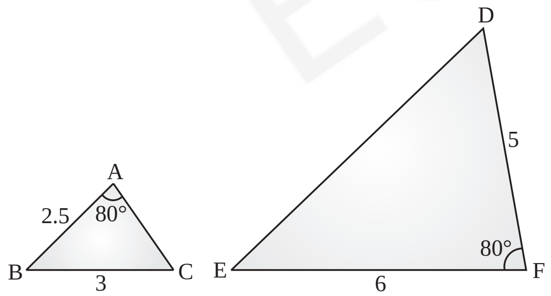
(iii)



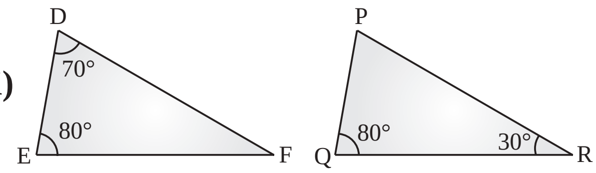
(iv)



(v)



(vi)



Sol. :(i) In ΔABC and ΔPQR,

$$\angle A = \angle P = 60°$$

$$\angle B = \angle Q = 80°$$

and $$\angle C = \angle R = 40°$$

∴ The corresponding angles of both the triangles are equal.
∴ By *AAA* similarity

$$\Delta ABC \sim \Delta PQR$$

(ii) In ΔABC and ΔPQR,

$$\frac{AB}{QR} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{BC}{PR} = \frac{2.5}{5} = \frac{1}{2}$$

and $$\frac{AC}{PQ} = \frac{3}{6} = \frac{1}{2}$$

Here, all the corresponding sides are in proportion.

∴ $\Delta ABC \sim \Delta QRP$ [By SSS Similarity]

(iii) In ΔLMP and ΔFED,

$$\frac{MP}{DE} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{LM}{EF} = \frac{2.7}{5}$$

$$\frac{LP}{DF} = \frac{3}{6} = \frac{1}{2}$$

i.e., $$\frac{MP}{DE} = \frac{LP}{DF} \neq \frac{LM}{EF}$$

Here, all the corresponding sides are not in proportion.
Hence, ΔLMP and ΔFED are not similar.

(iv) In ΔLMN and ΔRQP,

$$\angle M = \angle Q = 70°,$$

$$\frac{MN}{PQ} = \frac{2.5}{5} = \frac{1}{2}$$

and $$\frac{ML}{QR} = \frac{5}{10} = \frac{1}{2}$$

Here, two adjacent sides are in proportion and one angle is equal.

[By SAS Similarity]

$$\Delta LMN \sim \Delta RQP$$

(v) $\angle A$ is given in ΔABC, but the included side AC is not given.
∴ ΔABC and ΔDEF are not similar.

(vi) In ΔDEF,

We know that the sum of all the angles of a triangle is 180°.

∴ $\angle D + \angle E + \angle F = 180°$

$70° + 80° + \angle F = 180°$

$\angle F = 180° - 150° = 30°$

Similarly, in ΔPQR,

$\angle P + \angle Q + \angle R = 180°$

$\angle P + 80° + 30° = 180°$

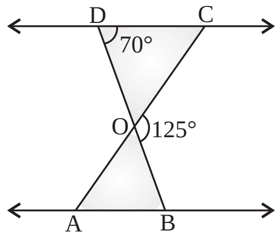
$\angle P = 180° - 110° = 70°$

Now, in $\angle DEF$ and ΔPQR,

$\angle D = \angle P = 70°$
$\angle E = \angle Q = 80°$
$\angle F = \angle R = 30°$

$\therefore \Delta DEF \sim \Delta PQR$ [By AAA Similarity]

2. In figure below, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 125°$ and $\angle CDO = 70°$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$.



Sol. : Here, DOB is a straight line.

$\therefore \angle DOC + 125° = 180°$

$\angle DOC = 180° - 125° = 55°$

Now, $\angle DCO + \angle CDO + \angle DOC = 180°$ [$\because$ We know that the sum of all the angles of a triangle is 180°.]

$\angle DCO + 70° + 55° = 180°$

$\Rightarrow \angle DCO = 180° - 125°$

$\angle DCO = 55°$

Now, $\Delta ODC \sim \Delta OBA$ [Given]

$\angle OCD = \angle OAB$

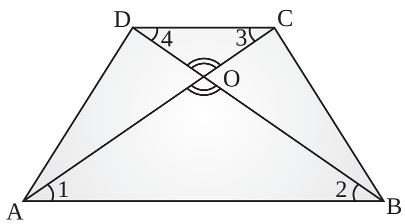
$\angle OAB = \angle DCO = 55°$

Hence, $\angle DOC = 55°$, $\angle DCO = 55°$ and $\angle OAB = 55°$

3. Diagonals AC and BD of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

Sol. :Given : Trapezium $ABCD$ whose diagonals are AC and BD, intersect each other at the point O and $AB \parallel DC$

To Prove : $\frac{OA}{OC} = \frac{OB}{OD}$



Proof : In ΔOCD and ΔOAB, $AB \parallel DC$ [Given,

$\angle 1 = \angle 3$, $\angle 2 = \angle 4$ [Alternate interior angle]

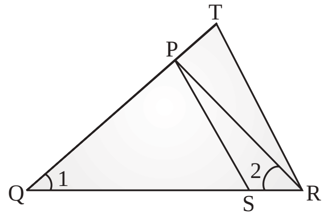
$\angle DOC = \angle BOA$ [Vertically Opposite angles]

$\therefore \Delta OCD \sim \Delta OAB$ [By AAS similarity]

$$\frac{OC}{OA} = \frac{OD}{OB}$$ [∵ The ratio of the corresponding sides of similar triangles are equal]

$$\therefore \quad \frac{OA}{OC} = \frac{OB}{OD}$$

4. In figure below, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\Delta PQS \sim \Delta TQR$.



Sol. : In ΔPQR, $\angle 1 = \angle 2$ [Given]

$PQ = PR$ ...(i)

[∵ Sides opposite to equal angles of a triangle are equal]

Given, $\frac{QR}{QS} = \frac{QT}{PR}$

$\frac{QR}{QS} = \frac{QT}{PQ}$ [From eqn. (i) $PR = PQ$]

or $\frac{QS}{QR} = \frac{PQ}{QT}$ ...(ii)

Now, in ΔPQS and ΔTQR

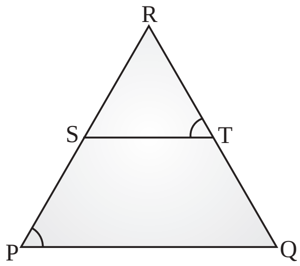
$\angle PQS = \angle TQR$ [Common angles]

and $\frac{QS}{QR} = \frac{PQ}{QT}$ [From eqn. (ii)]

$\therefore \quad \Delta PQS \sim \Delta TQR$ [By *SAS* similarity]

5. S and T are points on sides PR and QR of ΔPQR such that $\angle P = \angle RTS$. Show that $\Delta RPQ \sim \Delta RTS$.

Sol. : Points S and T lie on the sides PR and QR of ΔPQR such that $\angle P = \angle RTS$



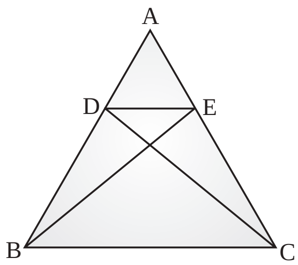
i.e., $\angle RPQ = \angle RTS$ [Given,

$\angle PRQ = \angle SRT$ [Common angles]

$\therefore \quad \Delta RPQ \sim \Delta RTS$ [By AA similarity]

Hence Proved

6. In figure below, if $\Delta ABE \cong \Delta ACD$, show that $\Delta ADE \sim \Delta ABC$.



Sol. : Given, $\Delta ABE \cong \Delta ACD$

$AB = AC$ and $AE = AD$ [By *CPCT*]

$\frac{AB}{AC} = 1$ and $\frac{AD}{AE} = 1$

$\therefore \quad \frac{AB}{AC} = \frac{AD}{AE}$ ...(i)

Now, in ΔADE and ΔABC,

$\frac{AD}{AE} = \frac{AB}{AC}$ [From eqn. (i)]

i.e., $\frac{AD}{AB} = \frac{AE}{AC}$

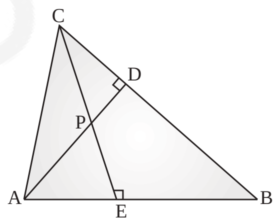
and $\angle DAE = \angle BAC$ [Each $\angle A$ is equal]

Hence, $\Delta ADE \sim \Delta ABC$ [By *SAS* similarity]

7. In figure below, altitudes *AD* and *CE* of $\triangle ABC$ intersect each other at the point *P*. Show that :

(i) $\Delta AEP \sim \Delta CDP$ **(ii) $\Delta ABD \sim \Delta CBE$**

(iii) $\Delta AEP \sim \Delta ADB$ **(iv) $\Delta PDC \sim \Delta BEC$**



Sol. : Given, The altitudes *AD* and *CE* intersect each others at the point *P*

(i) In ΔAEP and ΔCDP,

$\angle AEP = \angle CDP = 90°$

and $\angle APE = \angle CPD$ [Vertically Opposite angles]

$\therefore \quad \Delta AEP \sim \Delta CDP$ [By AA similarity]

(ii) In ΔABD and ΔCBE,

$\angle ADB = \angle CEB = 90°$

and $\angle ABD = \angle CBE$ [Common angles]

$\therefore \quad \Delta ABD \sim \Delta CBE$ [By AA similarity]

(iii) In ΔAEP and ΔADB,

$\angle AEP = \angle ADB = 90°$

and $\angle PAE = \angle DAB$ [Common angles]

$\therefore$ $\Delta AEP \sim \Delta ADB$ [By AA similarity]

(iv) In ΔPDC and ΔBEC,

$\angle PDC = \angle BEC = 90°$

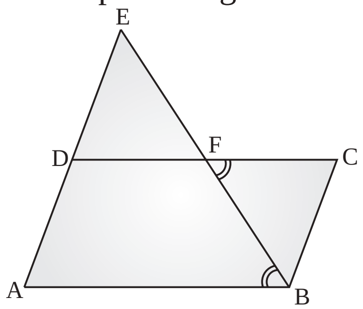
and $\angle PCD = \angle BCE$ [Common angles]

$\therefore$ $\Delta PDC \sim \Delta BEC$ [By AA similarity]

8. *E* is a point on the side *AD* produced of a parallelogram *ABCD* and *BE* intersects *CD* at *F*. Show that:

$\Delta ABE \sim \Delta CFB$.

Sol. : *E* is a point on the side *AD* produced of a parallelogram *ABCD* and *BE* intersects the side *CD* at *F*.



In Parallelogram *ABCD*,

$\angle A = \angle C$ ...(i)

[Corresponding angles are equal.]

Now, in ΔABE and ΔCFB,

$\angle EAB = \angle BCF$ [From eqn. (i)]

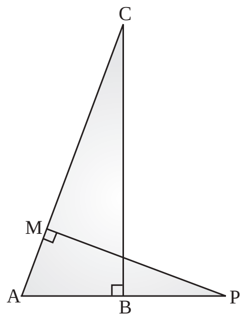
$\angle ABE = \angle CFB$

[Alternate interior angles $\because$ $AB \parallel FC$]

$\therefore$ $\Delta ABE \sim \Delta CFB$ [By *AA* similarity]

9. In figure below, *ABC* and *AMP* are two right triangles, right angled at *B* and *M* respectively. Prove that :

(i) $\Delta ABC \sim \Delta AMP$ **(ii) $\frac{CA}{PA} = \frac{BC}{MP}$**



Sol. :(i) In ΔABC and ΔAMP,

$\angle ABC = \angle AMP = 90°$ [Given]

$\angle BAC = \angle PAM$ [Common angle $\angle A$]

$\therefore$ $\Delta ABC \sim \Delta AMP$ [By AA similarity]

$\because$ $\Delta ABC \sim \Delta AMP$

$$\frac{AC}{AP} = \frac{BC}{MP}$$

[∵ The corresponding sides of similar triangles are in the same ratio.]

$\therefore \quad \frac{CA}{PA} = \frac{BC}{MP}$

Hence Proved

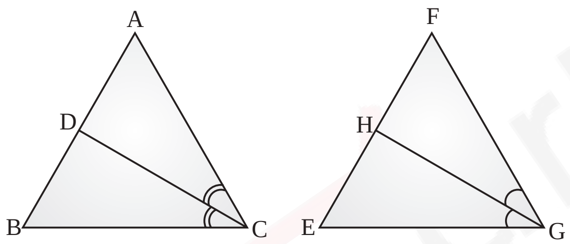
10. ***CD*** **and** ***GH*** **are respectively the bisectors of** $\angle ACB$ **and** $\angle EGF$ **such that** ***D*** **and** ***H*** **lie on sides** ***AB*** **and** ***FE*** **of** ΔABC **and** ΔEFG **respectively. If** $\Delta ABC \sim \Delta FEG$**, show that :**

(i) $\frac{CD}{GH} = \frac{AC}{FG}$ **(ii)** $\Delta DCB \sim \Delta HGE$

(iii) $\Delta DCA \sim \Delta HGF$

Sol. : In ΔABC and ΔFEG, CD and GH are the bisectors of $\angle ACB$ and $\angle EGF$ such that D lies on AB and H lies on FE.

$\Delta ABC \sim \Delta FEG$ [Given]



(i) In ΔACD and ΔFGH,

$\angle CAD = \angle GFH$ ...(i)

[∵ $\Delta ABC \sim \Delta FEG$]

$\angle ACB = \angle FGE$ [∵ $\Delta ABC \sim \Delta FEG$]

$\Rightarrow \quad \frac{1}{2}\angle ACB = \frac{1}{2}\angle FGE$

$\angle ACD = \angle FGH$ [Bisected angles]

By eqn. (i) and (ii),

$\Delta ACD \sim \Delta FGH$ [By AA Similarity]

$\therefore \quad \frac{CD}{GH} = \frac{AC}{FG}$

[∵ The corresponding sides of similar triangles are in the same ratio.]

(ii) In ΔDCB and ΔHGE,

$\angle DBC = \angle HEG$ ...(iii)

[∵ $\Delta ABC \sim \Delta FEG$]

$\angle ACB = \angle FGE$ [∵ $\Delta ABC \sim \Delta FEG$]

$\Rightarrow \quad \frac{1}{2}\angle ACB = \frac{1}{2}\angle FGE$

$\Rightarrow \quad \angle DCB = \angle HGE$ ...(iv)

[Bisected angles]

From eqn. (iii) and (iv),

$\Delta DCB \sim \Delta HGE$ [By AA similarity]

(iii) In ΔDCA and ΔHGF,

$\therefore \quad \angle DAC = \angle HFG$ ...(v) [$\because \Delta ABC \sim \Delta FEG$]

$\angle ACB = \angle FGE \quad [\because \Delta ABC \sim \Delta FEG]$

$\Rightarrow \quad \frac{1}{2}\angle ACB = \frac{1}{2}\angle FGE$

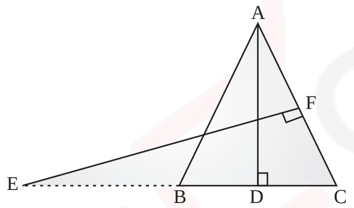
$\Rightarrow \quad \angle DCA = \angle HGF$ ...(vi)

[Bisected angles]

From eqn. (v) and (vi),

$\Delta DCA \sim \Delta HGF$ [By AA similarity]

11. In figure below, E is a point on side CB produced of an isosceles triangle ABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that $\Delta ABD \sim \Delta ECF$.



Sol. : ΔABC is an isosceles triangle in which $AB = BC$, $AD \perp BC$ and $EF \perp AC$,

Now, in ΔABC, $AB = AC \Rightarrow \angle ABC = \angle ACB$ ...(i)

[$\because$ Angle opposite to equal sides of a triangle are equal.]

In ΔADB and ΔEFC,

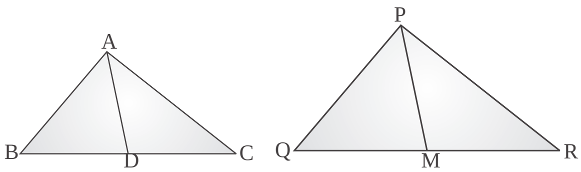
$\angle ABD = \angle ECF$ [From eqn. (i)]

and $\angle ADB = \angle EFC$ [Each 90°]

$\therefore \quad \Delta ADB \sim \Delta EFC$ या $\Delta ABD \sim \Delta ECF$

[By AA similarity]

12. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR. Show that $\Delta ABC \sim \Delta PQR$.



Sol. : Given, AD and PM are medians of ΔABC and ΔPQR respectively

To Prove : $\Delta ABC \sim \Delta PQR$

Proof : Given,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

$$\frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$$

or $$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

$\therefore \quad \Delta ADB \sim \Delta PMQ$ [By SSS Similarity]

Now, in ΔABC and ΔPQR,

$$\frac{AB}{PQ} = \frac{BC}{QR}$$ [Given]

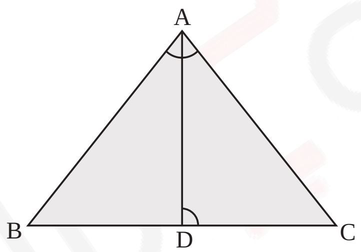
and $\angle B = \angle Q$ [$\because$ $\Delta ADB \sim \Delta PMQ$]

$\therefore \quad \Delta ABC \sim \Delta PQR$ [By SAS similarity]

13. ***D* is a point on the side *BC* of a triangle *ABC*, such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB.CD$.**

Sol. : In ΔABC and ΔDAC,

$\angle BAC = \angle ADC$ [Given]



and $\angle ACB = \angle DCA$ [Common angles]

$\therefore \quad \Delta ABC \sim \Delta DAC$ [By AA Similarity]

$$\frac{AC}{DC} = \frac{BC}{CA}$$

or $$\frac{AC}{CB} = \frac{CD}{CA}$$

or $$\frac{CA}{CB} = \frac{CD}{CA}$$

or $CA \times CA = CB \times CD$

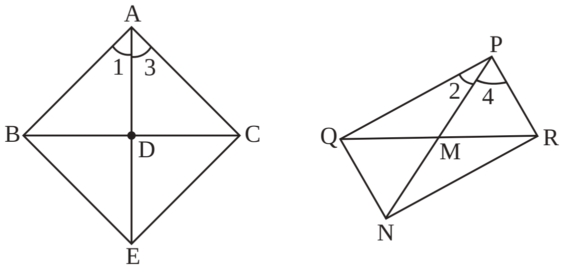
$\therefore \quad CA^2 = CB \times CD$ **Hence Proved**

14. **Sides *AB* and *AC* and median *AD* of a triangle *ABC* are respectively proportional to sides *PQ* and *PR* and median *PM* of another triangle *PQR*. Show that □*ABC* ~ □*PQR*.**

Sol. : Given : In ΔABC and ΔPQR,

$$\therefore \quad \frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \quad ...(i)$$

To Prove : $\Delta ABC \sim \Delta PQR$



Construction : Produce AD to point E such that $AD = DE$ and produce PM to point N such that $PM = MN$. Now join BE, CE, QN and RN.

Proof : Quadrilateral $ABEC$ and $PQNR$ are parallelogram, because their diagonals bisect each other at point D and M respectively.

$\therefore \quad BE = AC$

and $\quad QN = PR$

$$\frac{BE}{AC} = 1 \text{ and } \frac{QN}{PR} = 1$$

$$\frac{BE}{AC} = \frac{QN}{PR}$$

$$\frac{BE}{QN} = \frac{AC}{PR}$$

i.e., $\quad \dfrac{AB}{PQ} = \dfrac{BE}{QN}$ ...(ii)

From eqn. (i), $\quad \dfrac{AB}{PQ} = \dfrac{AD}{PM} = \dfrac{2AD}{2PM} = \dfrac{AE}{PN}$ ...(iii)

[Since, diagonals bisect each other]

From eqn. (ii) and (iii),

$$\frac{AB}{PQ} = \frac{BE}{QN} = \frac{AE}{PN}$$

$$\Delta ABE \sim \Delta PQN$$

$\angle 1 = \angle 2$ ...(iv)

Similarly,

$$\Delta ACE \sim \Delta PRN$$

$\angle 3 = \angle 4$ ...(v)

Adding eqn. (iv) and (v),

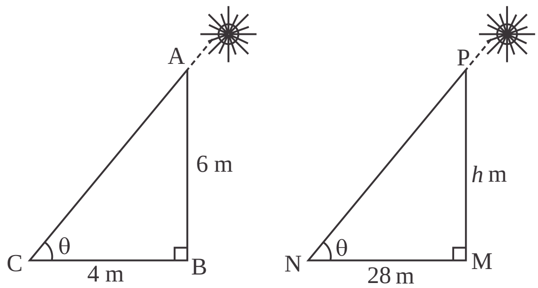
$$\angle 1 + \angle 3 = \angle 2 + \angle 4 \Rightarrow \angle BAC = \angle QPR$$

and $\quad \dfrac{AB}{PQ} = \dfrac{AC}{PR}$ [From eqn. (i)]

$\therefore \quad \Delta ABC \sim \Delta PQR$ [By SAS similarity]

15. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Sol. : Let AB is a vertical pole which is 6 m high and length of shadow on the ground of the pole is 4 m and makes an angle θ with the horizontal.



Again let PM be a tower whose height is h m and the length of the shadow of the tower on the ground is 28 m

i.e., $NM = 28$ m

In ΔABC and ΔPMN,

The rays of the sun will fall on the pole and the tower with the same angle.

$$\angle C = \angle N$$

$$\angle A = \angle P$$

and $\angle ABC = \angle PMN = 90°$

$\therefore \quad \Delta ABC \sim \Delta PMN$ [By AAA similarity]

Now, $\frac{AB}{PM} = \frac{BC}{MN} \Rightarrow \frac{AB}{BC} = \frac{PM}{MN}$

$$\frac{6}{4} = \frac{h}{28}$$

$\therefore \quad h = \frac{6 \times 28}{4} = 42$ m

Hence, the height of the tower is 42 m.

16. If AD and PM are medians of triangles ABC and PQR, respectively where $\Delta ABC \sim \Delta PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

Sol. : D and M are points on the sides B and QR of ΔABC and ΔPQR respectively such that AD and PM are the medians of ΔABC and ΔPQR.

Given, $\Delta ABC \sim \Delta PQR$

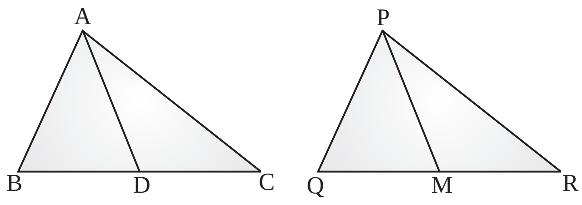
$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \quad ...(i)$$

and $\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$

$$BD = CD = \frac{1}{2}BC$$

$$QM = RM = \frac{1}{2}QR$$

[$\because$ D is the mid-point of BC and M is the mid-point of QR]



From eqn. (i), $\frac{AB}{PQ} = \frac{BC}{QR}$

$$\frac{AB}{PQ} = \frac{2BD}{2QM}$$

or $\frac{AB}{PQ} = \frac{BD}{QM}$

and $\angle ABD = \angle PQM$ [$\because \angle B = \angle Q$]

$\Delta ABD \sim \Delta PQM$ [By SAS similarity]

Hence, $\frac{AB}{PQ} = \frac{AD}{PM}$

Coordinate Geometry 7

NCERT SOLUTIONS

What's inside

– *In-Chapter Q's (solved)*

– *Textbook Exercise Q's (solved)*

Exercise – 7.1

1. Find the distance between the following pairs of points :
(i) (2, 3) and (4, 1) (ii) (– 5, 7) and (– 1, 3)
(iii) (*a*, *b*) and (– *a*, – *b*)

Sol. : **(i)** Let the given points are $A(2, 3)$ and $B(4, 1)$

Here, $x_1 = 2, y_1 = 3, x_2 = 4, y_2 = 1$

$$\therefore \quad AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4-2)^2 + (1-3)^2}$$
$$= \sqrt{2^2 + (-2)^2}$$
$$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \text{ units}$$

(ii) (– 5, 7) and (– 1, 3)

Let, the given points are $A(-5, 7)$ and $B(-1, 3)$

Here, $x_1 = -5, y_1 = 7, x_2 = -1, y_2 = 3$

$$\therefore \quad AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{\{(-1) - (-5)\}^2 + (3-7)^2}$$
$$= \sqrt{4^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32}$$
$$\therefore \quad AB = 4\sqrt{2} \text{ units}$$

(iii) (a, b) and ($-a$, $-b$)

Let, the given points are $A(a, b)$ and $B(-a, -b)$

Here, $x_1 = a, y_1 = b, x_2 = -a, y_2 = -b$

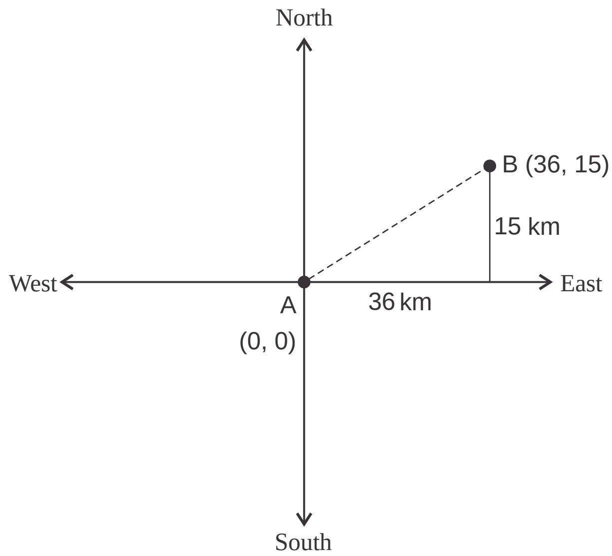
$$\therefore \quad AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-a-a)^2 + (-b-b)^2}$$
$$= \sqrt{(-2a)^2 + (-2b)^2}$$
$$= \sqrt{4a^2 + 4b^2} = \sqrt{4(a^2+b^2)}$$
$$\therefore \quad AB = 2\sqrt{a^2+b^2} \text{ units}$$

2. Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B? If A town B is located 36 cm east and 15 km north of the town A.

Sol. : Let given poitns are $A(0, 0)$ and $B(36, 15)$

Here, $x_1 = 0, y_1 = 0, x_2 = 36, y_2 = 15$

$$\therefore \quad AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(36-0)^2 + (15-0)^2}$$
$$= \sqrt{36^2 + 15^2} = \sqrt{1296+225}$$
$$= \sqrt{1521} = 39 \text{ units}$$



Given a town B is located 36 km east and 15 km north of town A.

Let, origin is A (0, 0) and point is B (36, 15)

Now, we have found the distance between points (0, 0) and (36, 15).

Hence, distance between town A and B = 39 km.

3. Determine if the points (1, 5), (2, 3) and (–2, –11) are collinear.

Sol. : Let, the given points are $A(1, 5)$, $B(2, 3)$ and $C(-2, -11)$

then, $AB = \sqrt{(2-1)^2 + (3-5)^2}$

$= \sqrt{1^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$ units

$BC = \sqrt{(-2-2)^2 + (-11-3)^2}$

$= \sqrt{(-4)^2 + (-14)^2}$

$= \sqrt{16+196} = \sqrt{212}$ units

$CA = \sqrt{(1+2)^2 + (5+11)^2}$

$= \sqrt{3^2 + 16^2} = \sqrt{9+256} = \sqrt{265}$ units

Now,

$AB + BC = \sqrt{5} + \sqrt{212}$

$= \sqrt{5} + 2\sqrt{53}$

and $AC = \sqrt{265}$

Hence, $AB + BC \neq AC$

Hence, given points are not collinear.

4. Check whether (5, –2), (6, 4) and (7, –2) are the vertices of an isosceles triangle.

Sol. : Let, the given points $A(5, -2)$, $B(6, 4)$ and $C(7, -2)$ then,

$AB = \sqrt{(6-5)^2 + [4-(-2)]^2}$

$= \sqrt{1^2 + 6^2} = \sqrt{1+36} = \sqrt{37}$

$BC = \sqrt{(7-6)^2 + (-2-4)^2}$

$= \sqrt{1^2 + (-6)^2} = \sqrt{1 + 36} = \sqrt{37}$

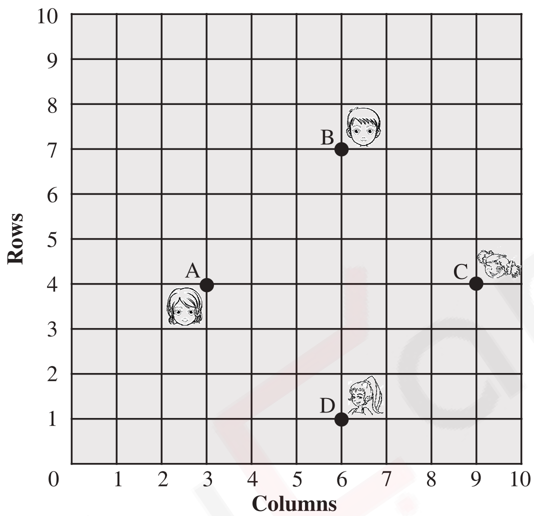
$AC = \sqrt{(7-5)^2 + (-2+2)^2}$

$= \sqrt{2^2 + 0^2} = \sqrt{4} = 2$

Here, $AB = BC = \sqrt{37}$ units

Hence, ΔABC is an isosceles triangle.

5. In a classroom, 4 friends are seated at the points *A*, *B*, *C* and *D* as shown in figure below. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli "Don't you think *ABCD* is square ?" Chameli disagrees. Using distance formula, find which of them is correct.



Sol. : In the given figure, the points *A*, *B*, *C* and *D* are (3, 4), (6, 7), (9, 4) and (6, 1) respectively.

$\therefore \quad AB = \sqrt{(6-3)^2 + (7-4)^2} = \sqrt{3^2 + 3^2}$

$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units

$BC = \sqrt{(9-6)^2 + (4-7)^2}$

$= \sqrt{3^2 + (-3)^2} = \sqrt{9+9}$

$= \sqrt{18} = 3\sqrt{2}$ units

$CD = \sqrt{(6-9)^2 + (1-4)^2}$

$= \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9}$

$= \sqrt{18} = 3\sqrt{2}$ units

$DA = \sqrt{(6-3)^2 + (1-4)^2}$

$= \sqrt{(3)^2 + (-3)^2} = \sqrt{9+9}$

$= \sqrt{18} = 3\sqrt{2}$ units

$\therefore \quad AB = BC = CD = DA$

Now, $AC = \sqrt{(9-3)^2 + (4-4)^2} = \sqrt{6^2 + 0^2}$

$= \sqrt{36+0} = \sqrt{36} = 6$ units

and $BD = \sqrt{(6-6)^2+(1-7)^2}$

$= \sqrt{0+(-6)^2} = \sqrt{36} = 6$ units

$\therefore$ $AC = BD$

$\therefore$ Four sides are equal and diagonals are also equal.

So, *ABCD* is a square.

Hence, Champa is correct.

6. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer :

(i) (– 1, – 2), (1, 0), (– 1, 2) and (– 3, 0)

(ii) (–3, 5), (3, 1), (0, 3) and (–1, 4)

(iii)(4, 5), (7, 6), (4, 3), and (1, 2)

Sol. : (i) Let, the given points are *A*(–1, –2), *B*(1, 0), *C*(–1, 2) and *D*(–3, 0), then,

$AB = \sqrt{(1+1)^2+(0+2)^2}$

$= \sqrt{2^2+2^2} = \sqrt{4+4}$

$= \sqrt{8} = 2\sqrt{2}$ units

$BC = \sqrt{(-1-1)^2+(2-0)^2}$

$= \sqrt{(-2)^2+2^2} = \sqrt{4+4}$

$= \sqrt{8} = 2\sqrt{2}$ units

$CD = \sqrt{(-3+1)^2+(0-2)^2}$

$= \sqrt{(-2)^2+(-2)^2} = \sqrt{4+4}$

$= \sqrt{8} = 2\sqrt{2}$ units

$DA = \sqrt{(-3+1)^2+(0+2)^2}$

$= \sqrt{(-2)^2+(2)^2} = \sqrt{4+4}$

$= \sqrt{8} = 2\sqrt{2}$ units

$\therefore$ $AB = BC = CD = DA$

Now, $AC = \sqrt{(-1+1)^2+(2+2)^2}$

$= \sqrt{0+4^2} = 4$ units

and $BD = \sqrt{(-3-1)^2+(0-0)^2}$

$= \sqrt{(-4)^2+0} = 4$ units

$\therefore$ $AC = BD$

All four sides *AB*, *BC*, *CD* and *DA* are equal and diagonals *AC* and *BD* are also equal. Hence, quadrilateral *ABCD* is a square.

(ii) Let, the given points are *A*(–3, 5), *B*(3, 1), *C*(0, 3) and *D*(–1, – 4).

$AB = \sqrt{(3+3)^2+(1-5)^2}$

$= \sqrt{(6)^2+(-4)^2}$

$= \sqrt{36+16}$

$= \sqrt{52} = 2\sqrt{13}$ units

$BC = \sqrt{(0-3)^2+(3-1)^2}$

$= \sqrt{(-3)^2 + (2)^2}$

$= \sqrt{9 + 4} = \sqrt{13}$ units

$CD = \sqrt{(-1-0)^2 + (-4-3)^2}$

$= \sqrt{(-1)^2 + (-7)^2}$

$= \sqrt{(1 + 49)}$

$= \sqrt{50} = 5\sqrt{2}$ units

$AD = \sqrt{(-1+3)^2 + (-4-5)^2}$

$= \sqrt{(2)^2 + (-9)^2}$

$= \sqrt{4 + 81} = \sqrt{85}$ units

$AC = \sqrt{(0+3)^2 + (3-5)^2}$

$= \sqrt{(3)^2 + (-2)^2}$

$= \sqrt{9 + 4} = \sqrt{13}$ unit

$BD = \sqrt{(-1-3)^2 + (-4-1)^2}$

$= \sqrt{(-4)^2 + (-5)^2}$

$= \sqrt{16 + 25} = \sqrt{41}$ units

Here, $BC + AC = AB$

Point A, B and C are collinear *i.e.*, all the points lie on same line.

Hence, $ABCD$ is not a quadrilateral.

(iii) Let, the given points are $P(4, 5)$, $Q(7, 6)$, $R(4, 3)$ and $S(1, 2)$.

$\therefore \quad PQ = \sqrt{(7-4)^2 + (6-5)^2}$

$= \sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$ units

$QR = \sqrt{(4-7)^2 + (3-6)^2}$

$= \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9}$

$= \sqrt{18} = 3\sqrt{2}$ units

$RS = \sqrt{(1-4)^2 + (2-3)^2}$

$= \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1}$

$= \sqrt{10}$ units

$SP = \sqrt{(4-1)^2 + (5-2)^2}$

$= \sqrt{3^2 + 3^2} = \sqrt{9+9}$

$= \sqrt{18} = 3\sqrt{2}$ units

$PR = \sqrt{(4-4)^2 + (3-5)^2}$

$= \sqrt{0 + (-2)^2} = 2$ units

$QS = \sqrt{(1-7)^2 + (2-6)^2}$

$= \sqrt{(-6)^2 + (-4)^2} = \sqrt{36+16}$

$= \sqrt{52} = 2\sqrt{13}$ units

$\therefore \quad PQ = RS$ and $QR = SP$

and $\quad PR \neq QS$

Hence, $PQRS$ is a parallelogram.

7. **Find the point on the *x*-axis which is equidistant from (2, –5) and (–2, 9).**

Sol. : ∵ The point lies on the *X*-axis

Hence, ordinate or *y*-coordinate = 0. Hence, point A (*x*, 0) lies on *X*-axis.

∵ Point *A*(*x*, 0) is equidistant from points *B*(2, – 5) and *C*(– 2, 9)

$\therefore \quad AB = AC$

$\sqrt{(2-x)^2 + (-5-0)^2} = \sqrt{(-2-x)^2 + (9-0)^2}$

$\Rightarrow (2-x)^2 + (-5-0)^2 = (-2-x)^2 + (9-0)^2$

$\Rightarrow 4 + x^2 - 4x + 25 = 4 + x^2 + 4x + 81$

$\Rightarrow \quad x^2 - 4x + 29 = x^2 + 4x + 85$

$\Rightarrow \quad -4x - 4x = 85 - 29$

$\Rightarrow \quad -8x = 56$

$\therefore \quad x = \frac{56}{-8} = -7$

Hence, the point lies on the *X*-axis is (–7, 0) which is equidistant from (2, – 5) and (– 2, 9).

8. **Find the values of *y* for which the distance between the points P(2, –3) and Q(10, *y*) is 10 units.**

Sol. : According to the question,

$PQ = 10$ units

$\sqrt{(10-2)^2 + [y-(-3)]^2} = 10$

$\Rightarrow \quad \sqrt{8^2 + (y+3)^2} = 10$

$\Rightarrow \quad \sqrt{64 + y^2 + 9 + 6y} = 10$

Squaring both the sides,

$73 + y^2 + 6y = 100$

$\Rightarrow y^2 + 6y + 73 - 100 = 0$

$\Rightarrow \quad y^2 + 6y - 27 = 0$

$\Rightarrow y^2 + 9y - 3y - 27 = 0$

$\Rightarrow y(y + 9) - 3(y + 9) = 0$

$\Rightarrow \quad (y + 9)(y - 3) = 0$

$\therefore \quad y = -9 \text{ or } y = 3$

9. **If *Q*(0, 1) is equidistant from *P*(5, –3) and *R*(*x*, 6), find the values of *x*. Also find the distances *QR* and *PR*.**

Sol. : ∵ Point *Q*(0, 1) is equidistant from *P*(5, – 3) and *R*(*x*, 6).

So, $\quad PQ = QR$

$\sqrt{(5-0)^2 + (-3-1)^2} = \sqrt{(x-0)^2 + (6-1)^2}$

$\Rightarrow (5-0)^2 + (-3-1)^2 = (x-0)^2 + (6-1)^2$

$\Rightarrow \quad (5)^2 + (-4)^2 = x^2 + 5^2$

$\Rightarrow \quad 25 + 16 = x^2 + 25$

$\Rightarrow \quad x^2 = 16 \Rightarrow x = \pm 4$

$\Rightarrow \quad x = -4, 4$

∴ Point *R* is either (4, 6) or (– 4, 6)

Condition-I : When point R is (4, 6), then

$$QR = \sqrt{(4-0)^2 + (6-1)^2}$$
$$= \sqrt{(4)^2 + (5)^2}$$
$$= \sqrt{16 + 25} = \sqrt{41} \text{ units}$$
$$PR = \sqrt{(4-5)^2 + (6+3)^2}$$
$$= \sqrt{(-1)^2 + (9)^2}$$
$$= \sqrt{1 + 81} = \sqrt{82} \text{ units}$$

Condition-II : When point R is $(-4, 6)$, then

$$QR = \sqrt{(-4-0)^2 + (6-1)^2}$$
$$= \sqrt{(-4)^2 + (5)^2}$$
$$= \sqrt{16 + 25} = \sqrt{41} \text{ units}$$
$$PR = \sqrt{(-4-5)^2 + (6+3)^2}$$
$$= \sqrt{(-9)^2 + (9)^2}$$
$$= \sqrt{81 + 81}$$
$$= \sqrt{162} = 9\sqrt{2} \text{ units}$$

10. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (–3, 4).

Sol. : Let, point $A(x, y)$, is equidistant from $B(3, 6)$ and $C(-3, 4)$.

$\therefore \quad AB = AC$

$\Rightarrow \quad \sqrt{(3-x)^2 + (6-y)^2} = \sqrt{(-3-x)^2 + (4-y)^2}$

$\Rightarrow 9 + x^2 - 6x + 36 + y^2 - 12y$

$= 9 + x^2 + 6x + 16 + y^2 - 8y$

$\Rightarrow x^2 + y^2 - 6x - 12y + 45 = x^2 + y^2 + 6x - 8y + 25$

$\Rightarrow x^2 + y^2 - 6x - 12y + 45 - x^2 - y^2 - 6x + 8y - 25 = 0$

$\Rightarrow \quad -12x - 4y + 20 = 0$

$\Rightarrow \quad -4(3x + y - 5) = 0$

$\Rightarrow \quad 3x + y - 5 = 0$

Exercise – 7.2

1. Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2 : 3.

Sol. : Let, the given points are $A(-1, 7)$ and $B(4, -3)$

Let, the coordinates of required point is (x, y)

Given, $m_1 = 2$ and $m_2 = 3$

By section formula,

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$
$$= \frac{2(4) + 3(-1)}{2 + 3} = \frac{8 - 3}{5}$$

$$= \frac{5}{5} = 1$$

and $$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

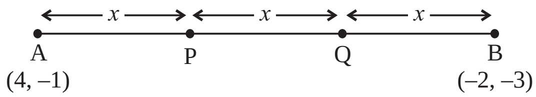
$$= \frac{2(-3) + 3(7)}{2+3} = \frac{-6+21}{5}$$

$$= \frac{15}{5} = 3$$

Thus, the coordinates of required point is (1, 3).

2. Find the coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).

Sol. : Let the points P and Q trisect the line segment joining the points A and B.



$$AP = PQ = QB = x$$
$$PB = PQ + QB = x + x = 2x$$
$$AQ = AP + PQ = x + x = 2x$$

$\therefore$ $AP : PB = x : 2x = 1 : 2$

and $AQ : QB = 2x : x = 2 : 1$

Since, point P divides AB in the ratio 1 : 2.

$\therefore$ Coordinates of P $$= \left(\frac{m_1x_2 + m_2x_1}{m_1+m_2}, \frac{m_1y_2 + m_2y_1}{m_1+m_2}\right)$$

$$= \left(\frac{1(-2) + 2(4)}{1+2}, \frac{1(-3) + 2(-1)}{1+2}\right)$$

$$= \left(\frac{-2+8}{3}, \frac{-3-2}{3}\right)$$

$$= \left(\frac{6}{3}, \frac{-5}{3}\right) = \left(2, \frac{-5}{3}\right)$$

and Q divides AB in the ratio 2 : 1.

$\therefore$ Coordinates of Q $$= \frac{m_1x_2 + m_2x_1}{m_1+m_2}, \frac{m_1y_2 + m_2y_1}{m_1+m_2}$$

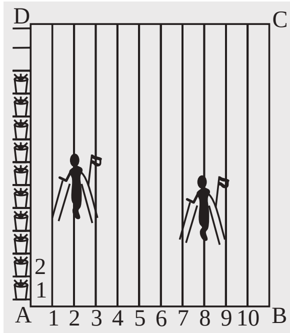
$$= \left(\frac{2(-2) + 1(4)}{2+1}, \frac{2(-3) + 1(-1)}{2+1}\right)$$

$$= \left(\frac{-4+4}{3}, \frac{-6-1}{3}\right)$$

$$= \left(0, \frac{-7}{3}\right)$$

Hence, the coordinates of the points of trisection of the line-segment are $P\left(2, \frac{-5}{3}\right)$ and $Q\left(0, \frac{-7}{3}\right)$.

3. To conduct sport Day activities, in your rectangular shaped school ground *ABCD,* lines have been drawn with chalk powder at a distance of 1m each. 100 flower pots have been placed at a distance of 1m from each other along *AD*, as shown in figure below. Niharika runs $\frac{1}{4}$ the the distance *AD* on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$th the distance *AD* on the eighth line and posts a red flag. What is the distance between both the flags ? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag ?

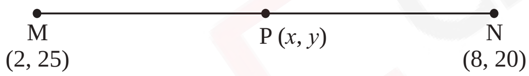


Sol. : From the given figure the position of green flag which is post by Niharika is $M\left(2, 100 \times \frac{1}{4}\right)$ *i.e.*, $M(2, 25)$ and the position of red flag which is post by Preet is $N\left(8, \frac{1}{5} \times 100\right)$ *i.e.*, $N(8, 20)$.

Hence, distance between flags,

$$MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(8-2)^2 + (20-25)^2}$$
$$= \sqrt{6^2 + (-5)^2}$$
$$= \sqrt{36 + 25}$$
$$= \sqrt{61} \text{ m}$$

Let, the position of the blue flag which is posted by Rashmi is $P(x, y)$ and it is at half the distance between M and N *i.e.*, P is the mid-point of MN.



Coordinates of $P(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$$= \left(\frac{2+8}{2}, \frac{25+20}{2}\right)$$
$$= \left(\frac{10}{2}, \frac{45}{2}\right)$$
$$= (5, 22.5)$$

Thus, the blue flag should be post on the 5th line and 22.5m above.

4. **Find the ratio in which the line segment joining the points (–3, 10) and (6, –8) is divided by (–1, 6)**

Sol. : Let the point $R(-1, 6)$ divides the line segment joining the points $P(-3, 10)$ and $Q(6, -8)$ in the ratio $k : 1$.

Coordinates of R

$$(-1, 6) = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$$

$$(-1, 6) = \frac{k \times 6 + 1 \times (-3)}{k+1}, \frac{k \times (-8) + 1 \times 10}{k+1}$$

$$(-1, 6) = \frac{6k - 3}{k+1}, \frac{-8k + 10}{k+1}$$

then $\quad -1 = \frac{6k-3}{k+1}$

$$6k - 3 = -k - 1$$
$$7k = 2$$
$$k = \frac{2}{7}$$
and $$6 = \frac{-8k + 10}{k + 1}$$
$$6k + 6 = -8k + 10$$
$$14k = 4$$
$$k = \frac{4}{14} = \frac{2}{7}$$
Hence, required ratio = 2 : 7

5. Find the ratio in which the line segment joining *A*(1, –5) and *B*(–4, 5) is divided by the *x*-axis. Also find the coordinates of the point of division.

Sol. : The points is on the *X*-axis. So, the ordinate point or the *y*-coordinate of the point is 0.

Hence, the coordinates of the point is $(x, 0)$

Now, let us consider the point $P(x, 0)$ divides the line segment joining the points $A(1, -5)$ and $B(-4, 5)$ in the ratio $k : 1$.

By section formula, coordinates of *P*

$$(x, 0) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$
$$(x, 0) = \left(\frac{-4k+1}{k+1}, \frac{5k-5}{k+1}\right)$$

Comparing the coordinates

$$0 = \frac{5k-5}{k+1} \Rightarrow 0 = 5k - 5$$
$$\Rightarrow \quad 5k = 5 \Rightarrow k = 1$$

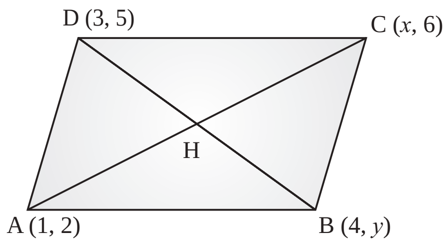
Hence, required ratio 1 : 1.

then, coordinates of the point of division

$$= \left(\frac{1\times(-4)+1\times 1}{1+1}, \frac{1\times 5+1\times(-5)}{1+1}\right)$$
$$= \left(\frac{-4+1}{2}, \frac{5-5}{2}\right) = \left(\frac{-3}{2}, 0\right)$$

6. If (1, 2), (4, *y*), (*x*, 6) and (3, 5) are the vertices of a parallelogram taken in order, find *x* and *y*.

Sol. : Let, $A(1, 2)$, $B(4, y)$, $C(x, 6)$ and $D(3, 5)$ are the vertices of a parallelogram.



Since, *ABCD* is a parallelogram. So, its diagonals *AC* and *BD* bisect each other.

Let, Mid point of *AC* and *BD* is *H*.

i.e., Mid point of AC = Mid point of BD

$$\left(\frac{1+x}{2}, \frac{2+6}{2}\right) = \left(\frac{4+3}{2}, \frac{y+5}{2}\right)$$

or $$\left(\frac{1+x}{2}, 4\right) = \left(\frac{7}{2}, \frac{y+5}{2}\right)$$

Comparing the coordinates

$$\frac{1+x}{2} = \frac{7}{2} \quad \Rightarrow 1 + x \quad = 7 \Rightarrow x = 6$$

and $$4 = \frac{5+y}{2} \Rightarrow 8 = 5 + y \Rightarrow y = 8 - 5 = 3$$

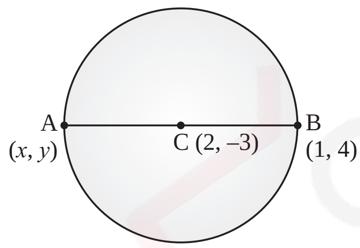
Hence, $x = 6$ and $y = 3$

7. Find the coordinates of a point *A*, where *AB* is the diameter of a circle whose centre is (2, –3) and *B* is (1, 4).

Sol. : Let AB is the diameter of circle whose centre is $C(2, -3)$ and the coordinates of B are (1, 4).

Let the coordinates of A are (x, y).

$\because$ AB is the diameter of the circle. So, C is the mid point of AB.



$\therefore$ Coordinates of mid point C = $\left(\frac{x+1}{2}, \frac{y+4}{2}\right)$ $\quad$ [$\because$ Coordinates of mid point = $\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$]

$$\boxed{(2-3) = \left(\frac{x+1}{2}, \frac{y+4}{2}\right)}$$

Comparing the coordinates :

Hence, $2 = \frac{x+1}{2}$ and $-3 = \frac{y+4}{2}$

or $4 = x + 1$ and $-6 = y + 4$

or $x = 4 - 1 = 3$ and $y = -6 - 4 = -10$

Hence, the coordinates of point A are (3, –10).

8. If *A* and *B* are (–2, –2) and (2, –4), respectively, find the coordinates of *P* such that $AP = \frac{3}{7}AB$ and *P* lies on the line segment *AB*.

Sol. : According to the question,

$$AP = \frac{3}{7}AB \Rightarrow \frac{AP}{AB} = \frac{3}{7}$$

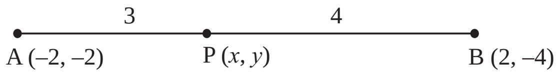
$$\frac{AP}{AP + PB} = \frac{3}{7}$$

$$\frac{AP+PB}{AP} = \frac{7}{3}$$

$$1 + \frac{PB}{AP} = \frac{7}{3}$$

$\therefore \quad \frac{PB}{AP} = \frac{4}{3} \Rightarrow \frac{AP}{PB} = \frac{3}{4}$

$\therefore \quad AP : PB = 3 : 4$



Now point $P(x, y)$ divided the line segment AB in the ratio 3 : 4.

$$\text{Coordinates of } P = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{3\times 2 + 4\times(-2)}{3+4}, \frac{3\times(-4) + 4\times(-2)}{3+4}\right)$$

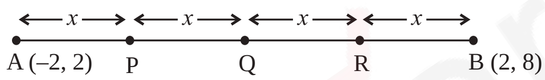
$$= \left(\frac{6-8}{7}, \frac{-12-8}{7}\right) = \left(\frac{-2}{7}, \frac{-20}{7}\right)$$

Hence, Coordinates of point P is $\left(\frac{-2}{7}, \frac{-20}{7}\right)$.

9. Find the coordinates of the points, which divide the line segment joining $A(-2, 2)$ and $B(2, 8)$ into four equal parts.

Sol. : Let the points P, Q and R lie on the line segment AB such that, $AP = PQ = QR = RB$

Let $AP = PQ = QR = RB = x$



Now, $\frac{AP}{PB} = \frac{x}{3x} = \frac{1}{3}$

Now P divides AB in the ratio 1 : 3.

$$\therefore \text{ Coordinates of } P = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{1(2) + 3(-2)}{1+3}, \frac{1(8) + 3(2)}{1+3}\right)$$

$$= \left(\frac{2-6}{4}, \frac{8+6}{4}\right) = \left(\frac{-4}{4}, \frac{14}{4}\right)$$

$$= \left(-1, \frac{7}{2}\right)$$

Point Q is the mid point of AB.

$$\therefore \text{Coordinates of } Q = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{-2+2}{2}, \frac{2+8}{2}\right) = \left(0, \frac{10}{2}\right)$$

$$= (0, 5)$$

Now, $\frac{AR}{RB} = \frac{3x}{x} = \frac{3}{1}$

Now R divides AB in the ratio 3 : 1.

$\therefore$Coordinates of Point R

$$= \left(\frac{3\times 2 + 1\times(-2)}{3+1}, \frac{3\times 8 + 1\times 2}{3+1}\right)$$

$$= \left(\frac{6-2}{4}, \frac{24+2}{4}\right)$$

$$= \left(\frac{4}{4}, \frac{26}{4}\right) = \left(1, \frac{13}{2}\right)$$

Here, required points are $\left(-1, \frac{7}{2}\right)$, (0, 5) and $\left(1, \frac{13}{2}\right)$.

10. Find the area of a rhombus if its vertices are (3, 0), (4, 5), (–1, 4) and (–2, –1) taken in order.

Sol. : Let, $A(3, 0)$, $B(4, 5)$, $C(-1, 4)$ and $D(-2, -1)$

The vertices of a rhombus

$$\therefore \text{Diagonal } AC = \sqrt{(-1-3)^2 + (4-0)^2}$$

$$= \sqrt{(-4)^2 + 4^2}$$

$$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2} \text{ units}$$

$$\text{and Diagonal } BD = \sqrt{(-2-4)^2 + (-1-5)^2}$$

$$= \sqrt{(-6)^2 + (-6)^2}$$

$$= \sqrt{36+36} = \sqrt{72} = 6\sqrt{2} \text{ units}$$

$$\therefore \text{Area of rhombus} = \frac{1}{2} \times \text{product of diagonals}$$

$$= \frac{1}{2} \times AC \times BD$$

$$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$$

$$= 24 \text{ sq. unit}$$

Coordinate Geometry 7

NCERT SOLUTIONS

What's inside

– In-Chapter Q's (solved)

– Textbook Exercise Q's (solved)

Introduction to Trigonometry

8

NCERT SOLUTIONS

What's inside

– *In-Chapter Q's (solved)*

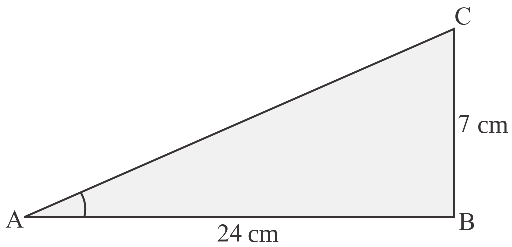
– *Textbook Exercise Q's (solved)*

Exercise – 8.1

1. In ΔABC, right-angled at B, AB = 24 cm BC = 7 cm. Determine :
(i) sin A, cos A **(ii) sin C, cos C**

Sol. : Given : In Right angle ΔABC,

$AB = 24$ cm, $BC = 7$ cm



By pythagoras theorem,

$$AC^2 = AB^2 + BC^2 = 24^2 + 7^2$$
$$= 576 + 49 = 625$$
$$\therefore \quad AC = \sqrt{625} = 25 \text{ cm}$$

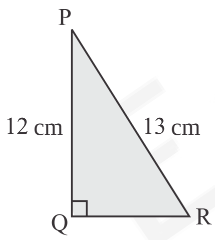
(i) $\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{7}{25}$

$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{24}{25}$

(ii) $\sin C = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{24}{25}$

$\cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{7}{25}$

2. In the given figure, find tan P – cot R.



Sol. : In Right angle ΔPQR,

$PQ = 12$ cm, $PR = 13$ cm

then, $PR^2 = PQ^2 + QR^2$ [By pythagoras theorem]

$$13^2 = 12^2 + QR^2$$
$$\Rightarrow \quad 169 = 144 + QR^2$$
$$\Rightarrow \quad QR^2 = 169 - 144$$
$$\Rightarrow \quad QR^2 = 25$$
$$\therefore \quad QR = \sqrt{25} = 5 \text{ cm}$$

then, $\tan P = \frac{\text{Perpendicular}}{\text{Base}} = \frac{QR}{PQ} = \frac{5}{12}$

and $\cot R = \frac{\text{Base}}{\text{Perpendicular}} = \frac{QR}{PQ} = \frac{5}{12}$

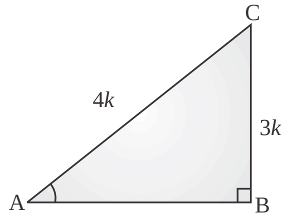
Hence, $\tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$

3. If sin A = $\frac{3}{4}$, calculate cos *A* and tan *A*.

Sol. : Given, $\sin A = \frac{3}{4}$

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{3}{4}$$

Let $BC = 3k$ and $AC = 4k$



By pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$\Rightarrow \quad (4k)^2 = AB^2 + (3k)^2$

$\Rightarrow \quad 16k^2 = AB^2 + 9k^2$

$\Rightarrow \quad AB^2 = 16k^2 - 9k^2 = 7k^2$

$\therefore \quad AB = \sqrt{7}k$

Now, $\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$$

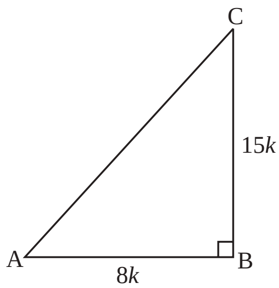
4. Given 15 cot *A* = 8, find sin *A* and sec *A*.

Sol. : Given, $15 \cot A = 8$

$\Rightarrow \quad \cot A = \frac{8}{15}$

$$\cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{BC} = \frac{8}{15}$$

Let $AB = 8k$, $BC = 15k$



By pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$= (8k)^2 + (15k)^2$$

$$= 64k^2 + 225k^2$$

$$= 289k^2$$

$\therefore \quad AC = \sqrt{289k^2} = 17k$

Now, $\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC}$

$= \frac{15k}{17k} = \frac{15}{17}$

$\sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{AB}$

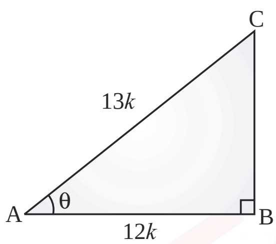
$= \frac{17k}{8k} = \frac{17}{8}$

5. Given $\sec \theta = \frac{13}{12}$, calculate all other trigonometric ratios.

Sol. : Given, $\sec \theta = \frac{13}{12}$

$\sec \theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{AB} = \frac{13}{12}$

Let, $AC = 13k$, $AB = 12\ k$



By pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$(13k)^2 = (12k)^2 + BC^2$$

$$\Rightarrow \quad 169k^2 = 144k^2 + BC^2$$

$$\Rightarrow \quad BC^2 = 169k^2 - 144k^2$$

$$= 25k^2$$

$\therefore \quad BC = \sqrt{25k^2} = 5k$

$\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC}$

$= \frac{5k}{13k} = \frac{5}{13}$

$\cos \theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC}$

$= \frac{12k}{13k} = \frac{12}{13}$

$\tan \theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB}$

$= \frac{5k}{12k} = \frac{5}{12}$

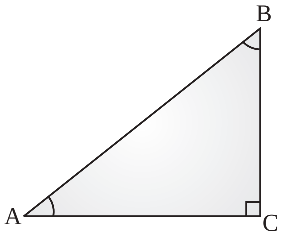
$\cot \theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{BC}$

$$= \frac{12k}{5k} = \frac{12}{5}$$

$$\text{cosec } \theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5}$$

6. If $\angle A$ and $\angle B$ are acute angles such that cos A = cos B, then show that $\angle A = \angle B$.

Sol. : Given, in ΔABC,



$$\cos A = \cos B$$

$$\therefore \quad \frac{AC}{AB} = \frac{BC}{AB}$$

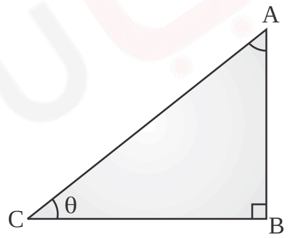
$$\Rightarrow \quad AC = BC$$

$\therefore \quad \angle B = \angle A$ [$\because$ Angles opposite to equal sides of a triangle are equals.]

7. If $\cot\theta = \frac{7}{8}$, evaluate :

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$, (ii) $\cot^2\theta$.

Sol. : Let, ΔABC is right-angled triangle at $\angle B$.



Given, $\cot\theta = \frac{7}{8}$

$$\frac{\text{Base}}{\text{Perpendicular}} = \frac{BC}{AB} = \frac{7}{8}$$

Let, BC = $7k$ and AB = $8k$

In ΔABC, By pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = (8k)^2 + (7k)^2$$

$$AC^2 = 64k^2 + 49k^2$$

$$AC = \sqrt{113k^2} = \sqrt{113}\,k$$

Now, $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$$= \frac{AB}{AC} = \frac{8k}{\sqrt{113}\,k} = \frac{8}{\sqrt{113}}$$

and $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$

$= \frac{BC}{AC} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$$= \frac{\left(1+\frac{8}{\sqrt{113}}\right)\left(1-\frac{8}{\sqrt{113}}\right)}{\left(1+\frac{7}{\sqrt{113}}\right)\left(1-\frac{7}{\sqrt{113}}\right)}$$

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{\left(1-\frac{64}{113}\right)}{\left(1-\frac{49}{113}\right)}$$

$$= \frac{113-64}{113-49} = \frac{49}{64}$$

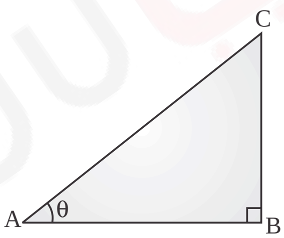
(ii) $\cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{7^2}{8^2} = \frac{49}{64}$

8. If 3 cot A = 4, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Sol. : Given, $3\cot A = 4$

$\Rightarrow \cot A = \frac{4}{3}$

$\frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{BC} = \frac{4}{3}$



Let $AB = 4k$, $BC = 3k$

In ΔABC, $AC^2 = AB^2 + BC^2$ [By pythagoras theorem]

$= (4k)^2 + (3k)^2$

$= 16k^2 + 9k^2$

$= 25k^2$

$\therefore AC = \sqrt{25k^2} = 5k$

$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$

$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$

$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$

$$\text{LHS} = \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2}$$

$$= \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}} = \frac{16-9}{16+9} = \frac{7}{25} \qquad \text{...(i)}$$

and RHS $= \cos^2 A - \sin^2 A$

$$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25} = \frac{16-9}{25} = \frac{7}{25} \qquad \text{...(ii)}$$

From eqn. (i) and eqn. (ii),

LHS = RHS

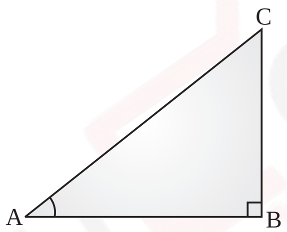
9. In triangle *ABC*, right-angled at *B*, if $\tan A = \frac{1}{\sqrt{3}}$, find the value of :

(i) sin *A* cos *C* + cos *A* sin *C*

(ii) cos *A* cos *C* – sin *A* sin *C*

Sol. : Given : $\tan A = \frac{1}{\sqrt{3}}$

$$\frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$$



Let $BC = k$, $AB = \sqrt{3}k$

In $\Delta ABC, AC^2 = AB^2 + BC^2$ [By pythagoras theorem]

$$= (\sqrt{3}k)^2 + k^2$$

$$= 3k^2 + k^2 = 4k^2$$

$\therefore \quad AC = \sqrt{4k^2} = 2k$

$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{3k}{2k} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{k}{2k} = \frac{1}{2}$$

(i) sin *A* cos *C* + cos *A* sin *C*

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

(ii) cos A cos C – sin A sin C

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

10. In Δ*PQR*, right angled at *Q*, *PR* + *QR* = 25 cm and *PQ* = 5 cm. Determine the values of sin *P*, cos *P* and tan *P*.

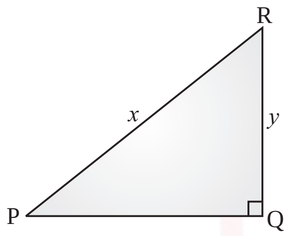
Sol. : Given : $PR + QR = 25$ cm

and $PQ = 5$ cm

Let $PR = x$ and $QR = y$

then $x + y = 25$

or $y = (25 - x)$



In Right angle ΔPQR,

$PR^2 = PQ^2 + QR^2$ [By pythagoras theorem]

$\Rightarrow$ $x^2 = 5^2 + y^2$

$\Rightarrow$ $x^2 = 5^2 + (25 - x)^2$

$\Rightarrow$ $x^2 = 25 + 625 + x^2 - 50x$

$\Rightarrow$ $50x = 650$

$x = \frac{650}{50} = 13$

$\therefore$ $PR = 13$ cm

and $QR = y = 25 - x = 25 - 13 = 12$ cm

$\therefore$ $\sin P = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{QR}{PR} = \frac{12}{13}$

$\cos P = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{PQ}{PR} = \frac{5}{13}$

$\tan P = \frac{\text{Perpendicular}}{\text{Base}} = \frac{QR}{PQ} = \frac{12}{5}$

11. State whether the following are true or false. Justify your answer.

(i) The value of tan *A* is always less than 1.

(ii) $\sec A = \frac{12}{5}$ **for some value of angle *A*.**

(iii) cos *A* is the abbreviation used for the cosecant of angle A.

(iv) cot *A* is the product of cot and *A*.

(v) sin θ = $\frac{4}{3}$ **for some angle θ.**

Sol. : (i) False, because the value of tan A increases from 0 to ∞ and tan 45° = 1, so the value of tan A

is not always less than 1.

(ii) True, because the value of sec A increases from 1 to ∞.

(iii) True, cos A is the short form for cosine of $\angle A$.

(iv) False, because cot A is the symbol of trigonometric ratio of angle. We cannot separate cot and A.

(v) False, because the value of sin θ from 0 to 1, but here $\sin\theta = \frac{4}{3}$ is greater than 1 which is not possible for sin θ.

Exercise – 8.2

1. Evaluate the following :

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

(ii) $\dfrac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°}$

(iii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

(iv) $\dfrac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

(v) $\dfrac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

Sol. : (i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

(ii) $$\frac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°} = \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + \frac{2}{1}}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2+2\sqrt{3}}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}} \times \frac{2\sqrt{2}-2\sqrt{6}}{2\sqrt{2}-2\sqrt{6}}$$

[Multiplying numerator and denominator by $2\sqrt{2}-2\sqrt{6}$]

$$= \frac{2\sqrt{6}-2\sqrt{18}}{(2\sqrt{2})^2-(2\sqrt{6})^2}$$

$$= \frac{2\sqrt{6}-6\sqrt{2}}{8-24}$$

$$= \frac{-2(3\sqrt{2}-\sqrt{6})}{-16} = \frac{3\sqrt{2}-\sqrt{6}}{8}$$

(iii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{3}{4} = 2$$

(iv) $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

$$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$$

$$= \frac{\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}} = \frac{3\sqrt{3}-4}{4+3\sqrt{3}}$$

[Multiplying numerator and denominator by $4-3\sqrt{3}$]

$$= \frac{3\sqrt{3}-4}{4+3\sqrt{3}} \times \frac{4-3\sqrt{3}}{4-3\sqrt{3}}$$

$$= \frac{12\sqrt{3} - 16 - 27 + 12\sqrt{3}}{(4)^2 - (3\sqrt{3})^2}$$

$$= \frac{24\sqrt{3} - 43}{16 - 27} = \frac{24\sqrt{3} - 43}{-11}$$

$$= \frac{43 - 24\sqrt{3}}{11}$$

(v) $\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\left(\frac{15+64-12}{12}\right)}{\left(\frac{1+3}{4}\right)} = \frac{67}{12} \times \frac{4}{4} = \frac{67}{12}$$

2. Choose the correct option and justify your choice :

(i) $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ} =$

(a) sin 60° **(b) cos 60°**

(c) tan 60° **(d) sin 30°**

(ii) $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ} =$

(a) tan 20° **(b) 1**

(c) sin 45° **(d) 0**

(iii) sin 2A = 2 sin A is true when A =

(a) 0° **(b) 30°**

(c) 45° **(d) 60°**

(iv) $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$ **=**

(a) cos 60° **(b) sin 60°**

(c) tan 60° **(d) sin 30°**

Sol. : (i) (a) $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ} = \frac{2\times\frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$

$= \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{3+1}{3}} = \frac{2}{\sqrt{3}}\times\frac{3}{4}$

$= \frac{\sqrt{3}}{2}$ = sin 60°

(ii) (b) $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ} = \frac{1-(1)^2}{1+(1)^2} = \frac{1-1}{1+1}$

$= \frac{0}{2} = 0$

(iii) (a) Given : sin 2A = 2 sin A

when, A = 0°

then,sin 2(0°) = 2 sin 0°

0 = 0 Right.

(iv)(c) $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ} = \frac{2\times\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$

$= \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{3-1}{3}} = \frac{2}{\sqrt{3}}\times\frac{3}{2}$

$= \sqrt{3}$ = tan 60°

3. If tan (A+B) = $\sqrt{3}$ and tan (A–B) = $\frac{1}{\sqrt{3}}$, 0 < A + B ≤ 90°; A > B, find A and B.

Sol. : Given :tan (A + B) = $\sqrt{3}$

⇒ tan (A + B) = tan 60°

⇒ $A + B$ = 60° ...(i)

and tan (A – B) = $\frac{1}{\sqrt{3}}$

⇒ tan (A – B) = tan 30°

$A - B = 30°$ ...(ii)

Adding on eqn. (i) and eqn. (ii),

$(A + B) + (A - B) = 60° + 30°$

$\Rightarrow \quad 2A = 90°$

$\Rightarrow \quad A = \frac{90°}{2} = 45°$

On putting the value of A in eqn. (ii),

$\Rightarrow \quad A - B = 30° \Rightarrow 45° - B = 30°$

$B = 45° - 30° = 15°$

Hence, $A = 45°$ and $B = 15°$

4. State whether the following are true or false. Justify your answer :

(i) sin (A + B) = sin A + sin B

(ii) The value of sin θ increases as θ increases.

(iii) The value of cos θ increases as θ increases.

(iv) sin θ = cos θ for all values of θ.

(v) cot A is not defined for A = 0°.

Sol. : (i) False, because :

when, $A = 60°$ and $B = 30°$

then,sin $(A + B)$= sin $(60° + 30°)$ = sin 90° = 1

andsin A + sin B= sin 60° + sin 30°

$= \frac{\sqrt{3}}{2} + \frac{1}{2}$

$= \frac{\sqrt{3}+1}{2}$

Hence, sin $(A + B) \neq$ sin A + sin B

(ii) True, because :

θ	0°	30°	45°	60°	90°
sin θ	0°	$\frac{1}{2}$	$\frac{1}{\sqrt{2}} = 0.707$	$\frac{\sqrt{3}}{2} = 0.866$	1

It is clear that the value of sin θ increases as θ increases

(iii) False

θ	0°	30°	45°	60°	90°
cos θ	1°	$\frac{\sqrt{3}}{2} = 0.866$	$\frac{1}{\sqrt{2}} = 0.707$	$\frac{1}{2} = 0.5$	0

It is clear that as θ increases, the value of cos θ decreases.

(iv) False, because only θ = 45°

$\sin\theta = \cos\theta = \frac{1}{\sqrt{2}}$

(v) True, because $A = 0°$ for $\tan A = \tan 0° = 0$

$\therefore \cot A = \frac{1}{\tan A} = \frac{1}{0} = \infty$ (undefined)

Exercise – 8.3

1. Express the trigonometric ratios sin *A*, sec *A* and tan *A* in terms of cot *A*.

Sol. : (i) We know that

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\Rightarrow \quad \text{cosec } A = \sqrt{1+\cot^2 A}$$

$$\therefore \quad \sin A = \frac{1}{\text{cosec } A} = \frac{1}{\sqrt{1+\cot^2 A}}$$

(ii) We know that

$$\sec^2 A = 1 + \tan^2 A$$

$$= 1+\frac{1}{\cot^2 A} = \frac{\cot^2 A+1}{\cot^2 A}$$

$$\sec A = \sqrt{\frac{\cot^2 A+1}{\cot^2 A}}$$

$$= \frac{\sqrt{1+\cot^2 A}}{\cot A}$$

(iii) $\tan A = \frac{1}{\cot A}$

2. Write all the other trigonometric ratio $\angle A$ in terms of sec *A*.

Sol. : (i) We know that,

$$\sin^2 A + \cos^2 A = 1$$

or $\quad \sin^2 A = 1 - \cos^2 A$

$$\sin A = \sqrt{1-\frac{1}{\sec^2 A}} = \sqrt{\frac{\sec^2 A-1}{\sec^2 A}}$$

$$\therefore \quad \sin A = \frac{\sqrt{\sec^2 A-1}}{\sec A}$$

(ii) $\cos A = \frac{1}{\sec A}$

(iii) We know that

$$\sec^2 A = 1 + \tan^2 A$$

or $\quad \tan^2 A = \sec^2 A - 1$

or $\quad \tan A = \sqrt{\sec^2 A-1}$

(iv) $\cot A = \frac{1}{\tan A} = \frac{1}{\sqrt{\sec^2 A-1}}$

(v) $\text{cosec } A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A-1}}$

3. Choose the correct option. Justify your choice.

(i) $9 \sec^2 A - 9 \tan^2 A =$

(a) 1 (b) 9 (c) 8 (d) 0

(ii) (1+ tan θ + sec θ) (1 + cot θ – cosec θ) =

(a) 0 (b) 1 (c) 2 (d) –1

(iii)(sec A + tan A) (1 – sin A) =

(a) sec A (b) sin A (c) cosec A (d) cos A

(iv) $\frac{1+\tan^2 A}{1+\cot^2 A}$ **=**

(a) $\sec^2 A$ (b) –1 (c) $\cos^2 A$ (d) $\tan^2 A$

Sol. : (i) (b) $9\sec^2 A - 9\tan^2 A = 9(\sec^2 A - \tan^2 A)$

$= 9 \times 1 = 9$ $\quad [\because \sec^2\theta - \tan^2\theta = 1]$

(ii)(c)(1 + tan θ + sec θ) (1 + cot θ – cosec θ)

$$= \left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right) \quad \left[\begin{array}{l}\because \tan\theta=\frac{\sin\theta}{\cos\theta}, \cot\theta=\frac{\cos\theta}{\sin\theta}\\ \sec\theta=\frac{1}{\cos\theta}, \text{cosec}\,\theta=\frac{1}{\sin\theta}\end{array}\right]$$

$$= \left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$$

$$= \left[\frac{(\sin\theta+\cos\theta)+1}{\cos\theta}\right]\left[\frac{(\sin\theta+\cos\theta)-1}{\sin\theta}\right]$$

$$= \frac{(\sin\theta+\cos\theta)^2-1}{\sin\theta\cos\theta} \quad [\because (a+b)(a-b)=a^2-b^2]$$

$$= \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$$

$$= \frac{1+2\sin\theta\cos\theta-1}{\sin\theta\cos\theta} \quad [\because \sin^2\theta+\cos^2\theta=1]$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2$$

(iii) (d)(sec A + tan A) (1 – sin A)

$$= \left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A) \quad \left[\begin{array}{l}\because \sec A=\frac{1}{\cos A}\\ \tan A=\frac{\sin A}{\cos A}\end{array}\right]$$

$$= \left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)$$

$$= \frac{1-\sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A}$$

$= \cos A$ $\quad [\because 1-\sin^2 A = \cos^2 A]$

(iv) (d) $\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A}{\text{cosec}^2 A}$ $\quad \left[\begin{array}{l}\because 1+\tan^2 A=\sec^2 A\\ 1+\cot^2 A=\text{cosec}^2 A\end{array}\right]$

$$= \frac{\left(\frac{1}{\cos^2 A}\right)}{\frac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

4. Prove the following identities, where the angles volved are acute angles for which the expressions are defined:

(i) $(\text{cosec}\ \theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

(ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

(iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\ \text{cosec}\ \theta$

(iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

(v) $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \text{cosec}\ A + \cot A$, using the identity $\text{cosec}^2 A = 1 + \cot^2 A$.

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

(vii) $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

(viii) $(\sin A + \text{cosec}\ A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

(ix) $(\text{cosec}\ A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A.$

Sol. : (i) LHS $= (\text{cosec}\ \theta - \cot\theta)^2$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 \qquad \left[\because \text{cosec}\ \theta = \frac{1}{\sin\theta},\ \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2 = \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} \qquad [\because \sin^2\theta = 1 - \cos^2\theta]$$

$$= \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} \qquad [\because a^2 - b^2 = (a-b)(a+b)]$$

$$= \frac{1-\cos\theta}{1+\cos\theta} = \text{RHS}$$

(ii) LHS $= \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$

$$= \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)(\cos A)}$$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{(\cos^2 A + \sin^2 A) + 1 + 2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{1+1+2\sin A}{(1+\sin A)(\cos A)} \qquad [\because \sin^2 A + \cos^2 A = 1]$$

$$= \frac{2+2\sin A}{(1+\sin A)(\cos A)} = \frac{2(1+\sin A)}{(1+\sin A)\cos A}$$

$$= \frac{2}{\cos A}$$

$= 2 \sec A$

$= \text{RHS}$

(iii) LHS= $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}$

$$= \frac{\left(\frac{\sin\theta}{\cos\theta}\right)}{\left(1-\frac{\cos\theta}{\sin\theta}\right)}+\left(\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}\right)$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$= \frac{\sin\theta}{\cos\theta}\times\frac{\sin\theta}{\sin\theta-\cos\theta}+\frac{\cos\theta}{\sin\theta}\times\frac{\cos\theta}{(\cos\theta-\sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta\,(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{\sin\theta\,(\cos\theta-\sin\theta)}$$

$$= \frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta\,(\sin\theta-\cos\theta)}$$

$$= \frac{(\sin\theta-\cos\theta)\,(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta\,(\sin\theta-\cos\theta)}$$

$$= \frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}=\frac{1}{\sin\theta\cos\theta}+\frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$= \operatorname{cosec}\theta\sec\theta + 1$

$= 1 + \sec\theta\operatorname{cosec}\theta = \text{RHS}$

(iv) LHS= $\dfrac{1+\sec A}{\sec A}=\dfrac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$

$$= \frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}}=\frac{\cos A+1}{\cos A}\times\frac{\cos A}{1}$$

$= \cos A + 1$ ...(i)

RHS = $\dfrac{\sin^2 A}{1-\cos A}$

$= \dfrac{1-\cos^2 A}{1-\cos A}=\dfrac{(1-\cos A)\,(1+\cos A)}{1-\cos A}$ $[\because \sin^2 A = 1 - \cos^2 A]$

$= 1 + \cos A$ ...(ii)

From eqn. (i) and (ii),

LHS = RHS

(v) LHS = $\dfrac{\cos A-\sin A+1}{\cos A+\sin A-1}$

Numerator and denominator dividing by $\sin A$

$$= \frac{\frac{\cos A}{\sin A}-\frac{\sin A}{\sin A}+\frac{1}{\sin A}}{\frac{\cos A}{\sin A}+\frac{\sin A}{\sin A}-\frac{1}{\sin A}}$$

$= \dfrac{\cot A-1+\operatorname{cosec} A}{\cot A+1-\operatorname{cosec} A}$ $\left[\because \frac{\cos A}{\sin A}=\cot A,\ \frac{1}{\sin A}=\operatorname{cosec} A\right]$

$= \dfrac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1}$

$= \dfrac{\cot A + \operatorname{cosec} A - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A - \operatorname{cosec} A + 1}$ [$\because$ $\operatorname{cosec}^2 A - \cot^2 A = 1$]

$= \dfrac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec} A - \cot A)(\operatorname{cosec} A + \cot A)}{\cot A - \operatorname{cosec} A + 1}$

$= \dfrac{(\cot A + \operatorname{cosec} A)[1 - (\operatorname{cosec} A - \cot A)]}{\cot A - \operatorname{cosec} A + 1}$

$= \dfrac{(\cot A + \operatorname{cosec} A)(1 - \operatorname{cosec} A + \cot A)}{\cot A - \operatorname{cosec} A + 1}$

$= \cot A + \operatorname{cosec} A = \operatorname{cosec} A + \cot A$

= RHS

(vi) LHS $= \sqrt{\dfrac{1+\sin A}{1-\sin A}}$

$= \sqrt{\dfrac{1+\sin A}{1-\sin A}} \times \sqrt{\dfrac{1+\sin A}{1+\sin A}}$ [$\because$ Multiplying by $\sqrt{1+\sin A}$ in numerator and denominator]

$= \dfrac{1+\sin A}{\sqrt{1-\sin^2 A}} = \dfrac{1+\sin A}{\sqrt{\cos^2 A}}$ [$\because$ $1 - \sin^2 A = \cos^2 A$]

$= \dfrac{1+\sin A}{\cos A} = \dfrac{1}{\cos A} + \dfrac{\sin A}{\cos A}$

$= \sec A + \tan A =$ RHS

(vii) LHS $= \dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$= \dfrac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$

$= \dfrac{\sin\theta[1 - 2(1 - \cos^2\theta)]}{\cos\theta[(2\cos^2\theta - 1)]}$ [$\because$ $\sin^2\theta + \cos^2\theta = 1$]

$= \dfrac{\sin\theta(1 - 2 + 2\cos^2\theta)}{\cos\theta(2\cos^2\theta - 1)} = \dfrac{\sin\theta(2\cos^2\theta - 1)}{\cos\theta(2\cos^2\theta - 1)}$

$= \dfrac{\sin\theta}{\cos\theta} = \tan\theta =$ RHS

(viii) LHS $= (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A$

$= (\sin^2 A + \cos^2 A) + 4 + \operatorname{cosec}^2 A + \sec^2 A$

$= 1 + 4 + (1 + \cot^2 A) + (1 + \tan^2 A)$ [$\because$ $\operatorname{cosec}^2 A = 1 + \cot^2 A$, $\sec^2 A = 1 + \tan^2 A$]

$= 7 + \cot^2 A + \tan^2 A$

$= 7 + \tan^2 A + \cot^2 A = \text{LHS}$

(ix) LHS = (cosec A – sin A) (sec A – cos A)

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)$$

$$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} = \cos A \sin A \qquad \text{...(i)}$$

$$\text{Now, RHS} = \frac{1}{\tan A + \cot A}$$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$

$$= \frac{1}{\sin^2 A + \cos^2 A} \times \frac{\sin A \cos A}{1}$$

$$= \frac{1}{1} \times \sin A \cos A$$

$$= \sin A \cos A \qquad \text{...(ii)}$$

From eqn. (i) and (ii),

LHS = RHS

(x) $\text{LHS} = \dfrac{1+\tan^2 A}{1+\cot^2 A}$

$$= \frac{1+\tan^2 A}{1+\frac{1}{\tan^2 A}} = \frac{1+\tan^2 A}{\frac{\tan^2 A + 1}{\tan^2 A}}$$

$$= (1+\tan^2 A) \times \frac{\tan^2 A}{1+\tan^2 A}$$

$$= \tan^2 A = \text{RHS}$$

and $\left(\dfrac{1-\tan A}{1-\cot A}\right)^2 = \left(\dfrac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$ $\qquad \left[\because \cot A = \dfrac{1}{\tan A}\right]$

$$= \left(\frac{1-\tan A}{\frac{\tan A - 1}{\tan A}}\right)^2 = \left(1-\tan A \times \frac{\tan A}{\tan A - 1}\right)^2$$

$= (-\tan A)^2 = \tan^2 A).$

Some Applications of Trigonometry

9

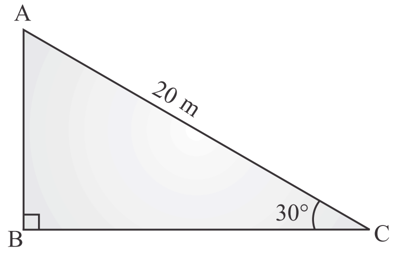
NCERT SOLUTIONS

What's inside

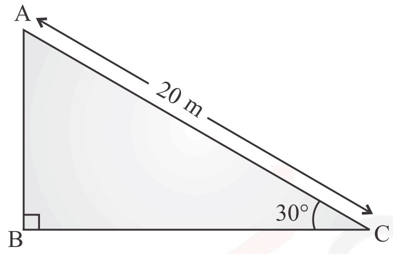
- In-Chapter Q's (solved)
- Textbook Exercise Q's (solved)

Exercise – 9.1

1. **A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30° (See Figure Below).**



Sol. : In the given figure, AB is the height of the pole and the length of the rope is 20 m, which is tied at the top of the pole.



In right angled ΔABC,

$$\sin 30° = \frac{AB}{AC}$$

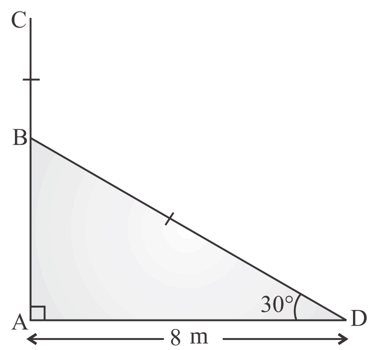
$$\frac{1}{2} = \frac{AB}{20}$$

$$AB = \frac{20}{2} = 10 \text{ m}$$

Hence, the height of the pole is 10 m.

2. **A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.**

Sol. : Let AC be the height of the tree, which is broken from B due to storm and the broken part BC touches the ground at point D making an angle of 30° with the ground.



Given : $AD = 8$ m

$\angle BDA = 30°$

In ΔABD,

$$\tan 30° = \frac{AB}{AD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{8}$$

$$\Rightarrow \quad AB = \frac{8}{\sqrt{3}}$$

Again, in ΔABD,

$$\cos 30° = \frac{AD}{BD}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{8}{BD}$$

$$\Rightarrow \quad BD = \frac{8 \times 2}{\sqrt{3}} = \frac{16}{\sqrt{3}} \text{ m}$$

$$AC = AB + BC$$

$$\therefore \quad AC = \frac{8}{\sqrt{3}} + BD \qquad [\because BC = BD]$$

$$\Rightarrow \quad AC = \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}}$$

$$= \frac{24}{\sqrt{3}} = 8\sqrt{3} \text{ m}$$

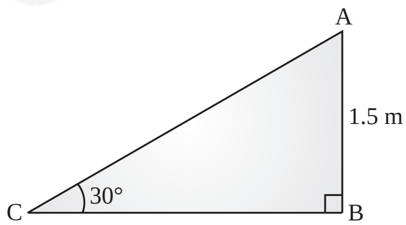
Hence, the height of the tree is $8\sqrt{3}$ m.

3. **A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, he prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, he wants to have a steep slide at a height of 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?**

Sol. : **(i)** For children below the age of 5 years :

Let the slide be AC.

The height of the slide AB = 1.5 m and the slide AC makes an angle of 30° with the ground.



In right angle ΔABC,

$$\sin 30° = \frac{AB}{AC}$$

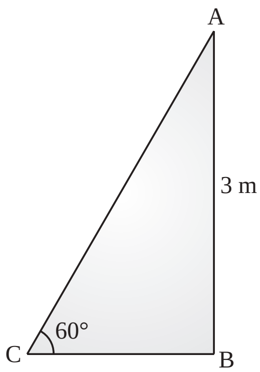
$$\Rightarrow \quad \frac{1}{2} = \frac{1.5}{AC}$$

$$\therefore \quad AC = 1.5 \times 2 = 3 \text{ m}$$

(ii) For children above the age of 5 years :

Let the slide be AC. The height of the slide AB = 3 m and the slide AC makes an angle of 60° with the ground.

In right angled ΔABC,



$$\sin 60° = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{3}{AC}$$

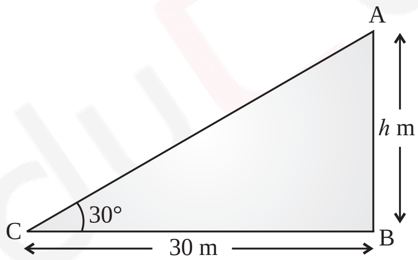
$$\therefore \quad AC = \frac{3 \times 2}{\sqrt{3}}$$

$$= 2\sqrt{3} \text{ m}$$

4. **The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.**

Sol. : Let there is a tower *AB* of height *h* metres, which is at a distance of 30 metres from a point *C* on the ground.

The angle of elevation of the top of the tower from point *C* is 30°.



Then, in right angle ΔABC,

$$\tan 30° = \frac{AB}{BC}$$

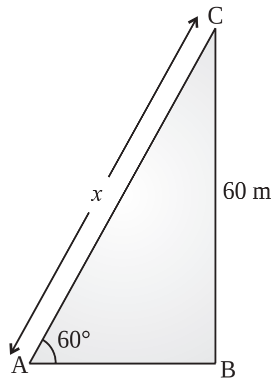
$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{30}$$

$$\Rightarrow \quad h = \frac{30}{\sqrt{3}} = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\therefore \quad h = \frac{30\sqrt{3}}{3} = 10\sqrt{3} \text{ m}$$

5. **A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.**

Sol. : Let the string *AC* be of length *x* m and the kite is flying at a height *BC* = 60 m from the ground and the string is temporarily tied at point *A*. The string of the kite makes an angle of 60° with the ground.



In right angled ΔABC,

$$\sin 60° = \frac{BC}{AC}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{60}{x}$$

$$\Rightarrow \quad x = \frac{60 \times 2}{\sqrt{3}} = \frac{120}{\sqrt{3}}$$

$$\therefore \quad x = \frac{120}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{120\sqrt{3}}{3} = 40\sqrt{3} \text{ m}$$

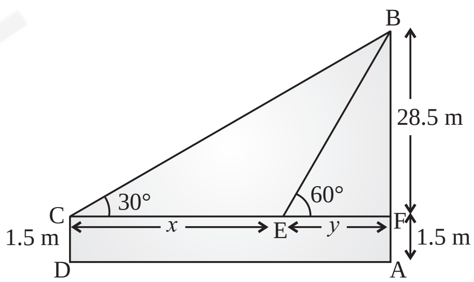
Hence, the length of string be $40\sqrt{3}$ m.

6. **A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.**

Sol. : Given : Height of building, $AB = 30$ m

and height of the boy $CD = 1.5$ m

Let the boy's eye be C and the angle of elevation of the top of the building from the eye are 30° and 60°. Let he walked x metres towards the building.



So, $AB = 30$ m

$BF = AB - AF = 30 - 1.5$

$= 28.5$ m

$CE = x$ and $EF = y$

In right angled ΔBCF,

$$\tan 30° = \frac{BF}{CF}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{28.5}{x + y}$$

$\Rightarrow \quad x + y = 28.5\sqrt{3}$ ...(i)

In right angled ΔBEF,

$$\tan 60° = \frac{BF}{EF}$$

$$\Rightarrow \quad \sqrt{3} = \frac{28.5}{y}$$

$$\Rightarrow \quad y = \frac{28.5}{\sqrt{3}} \quad \text{...(ii)}$$

Put the value of y in equation (i),

$$x + \frac{28.5}{\sqrt{3}} = 28.5\sqrt{3}$$

$$\Rightarrow \quad x = 28.5\sqrt{3} - \frac{28.5}{\sqrt{3}}$$

$$= \frac{28.5\sqrt{3} \times \sqrt{3} - 28.5}{\sqrt{3}}$$

$$= \frac{28.5\,(3-1)}{\sqrt{3}}$$

$$= \frac{28.5 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

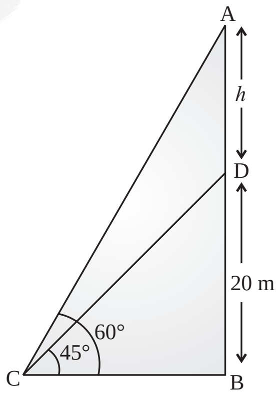
$$= \frac{28.5 \times 2\sqrt{3}}{3} = 9.5 \times 2\sqrt{3}$$

$$\Rightarrow \quad x = 19\sqrt{3} \text{ m}$$

Hence, the boy walked $19\sqrt{3}$ metres towards the building.

7. **From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.**

Sol. : Let the height of the transmission tower $AD = h$ m. The height of the building $BD = 20$ m and the tower fixed at the top of the building. From a point C on the ground, the angles of elevation of the bottom D and top A of the tower are 45° and 60° respectively.



Now, in right angled ΔDBC,

$$\tan 45° = \frac{BD}{BC}$$

$$\Rightarrow \quad 1 = \frac{20}{BC}$$

$\therefore \quad BC = 20$ m ...(i)

In right angled ΔABC,

$$\tan 60° = \frac{AB}{BC} = \frac{AD+BD}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h+20}{20} \quad \text{[From eqn. (i) } BC = 20\text{]}$$

$$\Rightarrow \quad 20\sqrt{3} = h + 20$$

$$\therefore \quad h = 20\sqrt{3} - 20$$

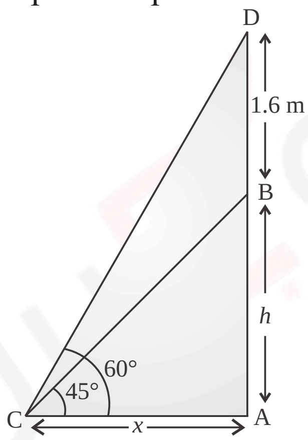
$$= 20(\sqrt{3}-1) \text{ m}$$

Hence, the height of transmission tower is $20(\sqrt{3}-1)$m.

8. **A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.**

Sol. : Let the pedestal is AB whose height is h metres. A statue, BD = 1.6 m tall stands on the top of the pedestal.

Let a point C on the ground from where the angle of elevation of the top of the statue is 60° and the angle of elevation of the top of the pedestal is 45°.



Let, $AC = x$

In right angled ΔABC,

$$\tan 45° = \frac{AB}{AC}$$

$$\Rightarrow \quad 1 = \frac{h}{x}$$

$$\therefore \quad x = h \qquad \text{...(i)}$$

In right angled ΔDAC,

$$\tan 60° = \frac{DA}{AC} = \frac{AB+BD}{AC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h+1.6}{x}$$

$$\Rightarrow \quad \sqrt{3}x = h + 1.6$$

Substituting the value of x from eqn. (i), we get

$$\sqrt{3}h = h + 1.6$$

$$\Rightarrow \quad \sqrt{3}h - h = 1.6$$

$$\Rightarrow \quad h(\sqrt{3}-1) = 1.6$$

$$\Rightarrow \quad h = \frac{1.6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{1.6(\sqrt{3}+1)}{3-1}$$

$$= \frac{1.6}{2}(\sqrt{3}+1)$$

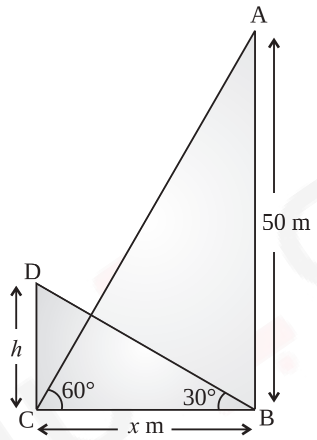
$$\therefore \quad h = 0.8(\sqrt{3}+1)$$

Hence, the height of pedestal = $0.8(\sqrt{3}+1)$ m

9. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

Sol. : Let the building is *CD*, whose height is *h* metres.

The tower *AB* = 50 m high. The angle of elevation of the top of the building from the foot of the tower is $\angle CBD = 30°$ and the angle of elevation of the top of the tower from the foot of building is 60°.



Let, $BC = x$ m

In right angled ΔABD,

$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{50}{x}$$

$$\Rightarrow \quad x = \frac{50}{\sqrt{3}} \quad \text{...(i)}$$

In right angled ΔBCD,

$$\tan 30° = \frac{CD}{BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow \quad x = h\sqrt{3}$$

Substituting the value of *x* from eqn. (i), we get

$$\frac{50}{\sqrt{3}} = h\sqrt{3}$$

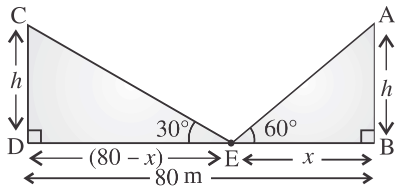
$$\Rightarrow \quad 50 = h\sqrt{3} \times \sqrt{3} = 3h$$

$$\therefore \quad h = \frac{50}{3} = 16\frac{2}{3} \text{ m}$$

Hence, the height of the building is $16\frac{2}{3}$ m.

10. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.

Sol. : Let *AB* and *CD* be two poles of equal length placed on either side of a road 80 m wide. Let the angles of elevation of the top of the poles from a point *E* on the road be 60° and 30°.



i.e., $\angle AEB = 60°, \angle CED = 30°$

Let, $AB = CD = h$ m,

and $BE = x$ m

then $DE = (80 - x)$ m

In right angled ΔABE,

$$\tan 60° = \frac{AB}{BE}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow h = \sqrt{3}x \quad ...(i)$$

In right angled ΔCDE,

$$\tan 30° = \frac{CD}{DE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{80 - x}$$

$$\Rightarrow 80 - x = \sqrt{3}h \quad ...(ii)$$

On putting the value of *h* from eqn. (i) into eqn. (ii), we get

$$80 - x = \sqrt{3}(\sqrt{3}x)$$

$$\Rightarrow 80 - x = 3x$$

$$\Rightarrow 80 = 3x + x = 4x$$

$$\therefore x = \frac{80}{4} = 20 \text{ m}$$

On putting the value of *x* in eqn. (i),

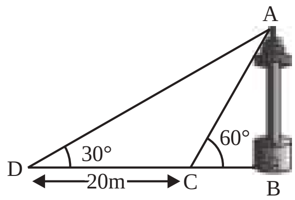
$$h = \sqrt{3}x = \sqrt{3} \times 20$$

$$= 20\sqrt{3} \text{ m}$$

and $DE = 80 - x = 80 - 20 = 60$ m

Hence, the height of poles is $20\sqrt{3}$ m. The distance between the poles are 20 m and 60 m respectively.

11. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal.



Sol. : Let, the height of the tower AB be h m and x m be the width of the canal BC.

In right angled ΔABC,

$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad h = \sqrt{3}x \quad ...(i)$$

In right angled ΔABD,

$$\tan 30° = \frac{AB}{BD} = \frac{AB}{BC + CD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x+20}$$

$$\Rightarrow \quad x + 20 = h\sqrt{3} \quad ...(ii)$$

Putting the value of h in eqn. (ii),

$$x + 20 = x\sqrt{3} \times \sqrt{3}$$

$$\Rightarrow \quad x + 20 = 3x$$

$$\Rightarrow \quad 20 = 3x - x$$

$$\Rightarrow \quad 2x = 20 \Rightarrow x = \frac{20}{2} = 10 \text{ m}$$

On putting the value of x in eqn. (i),

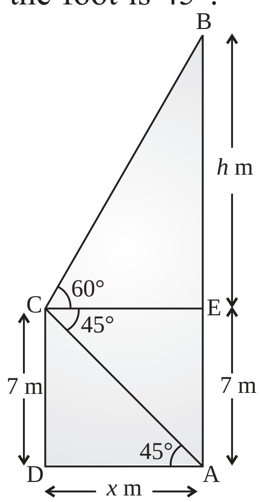
$$h = \sqrt{3}x = \sqrt{3} \times 10$$

$$\therefore \quad h = 10\sqrt{3} \text{ m}$$

Hence, the height of the tower is $10\sqrt{3}$ m and the width of the canal is 10 m.

12. **From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.**

Sol. : Given that the angle of elevation of the top of only tower AB from a 7m high building CD is 60° and the angle of depression of the foot is 45°.



i.e., $\quad \angle BCE = 60°$

and $\angle ACE = 45°$

$\therefore$ $CE \parallel AD$

then $\angle ACE = \angle CAD$

$= 45°$ [Alternate internal angle]

Let, the distance between the tower and the building be $AD = x$ metres.

In right angle ΔCDA,

$$\tan 45° = \frac{CD}{AD}$$

$$\Rightarrow \quad 1 = \frac{7}{x}$$

$$\Rightarrow \quad x = 7 \text{ m}$$

In right angle ΔBEC,

$$\tan 60° = \frac{BE}{CE}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x} \quad [\because AD = CE]$$

$$\Rightarrow \quad h = \sqrt{3}x$$

$$\Rightarrow \quad h = \sqrt{3} \times 7$$

$$\therefore \quad h = 7\sqrt{3} \text{ m}$$

Hence, height of tower $AB = AE + BE$

$= 7 + h$

$= 7 + 7\sqrt{3}$

$= 7(1+\sqrt{3})$ m

13. **As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.**

Sol. : The height of the lighthouse from sea level AC is $CD = 75$ m.

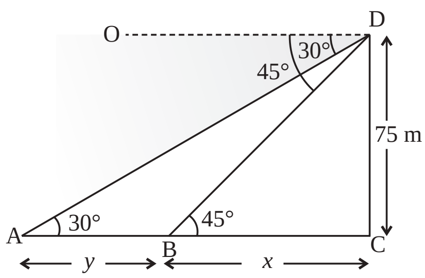
Let A and B be the positions of two ships on the sea level. Let the distance between two ships A and B be $AB = 5$ m.

The angles of depression of the two ships from the top D of the lighthouse are $\angle ODA = 30°$ and $\angle ODB = 45°$.

Here, $\angle DAC = \angle ODA = 30°$

and $\angle CBD = \angle ODB = 45°$

Let, $BC = x$



In right angle ΔDBC,

$$\tan 45° = \frac{CD}{BC}$$

$\Rightarrow \quad 1 = \frac{75}{x}$

$\therefore \quad x = 75$ m

In right angle ΔACD,

$$\tan 30° = \frac{CD}{AC} = \frac{CD}{AB + BC}$$

$$\frac{1}{\sqrt{3}} = \frac{75}{y + x}$$

$$x + y = 75\sqrt{3}$$

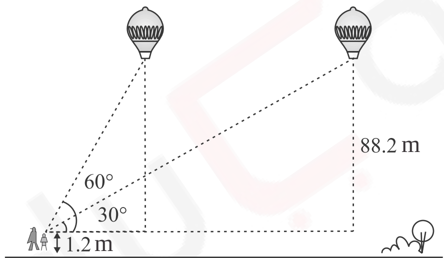
$\Rightarrow \quad 75 + y = 75\sqrt{3}$

$\Rightarrow \quad y = 75\sqrt{3} - 75$

$\therefore \quad y = 75(\sqrt{3} - 1)$

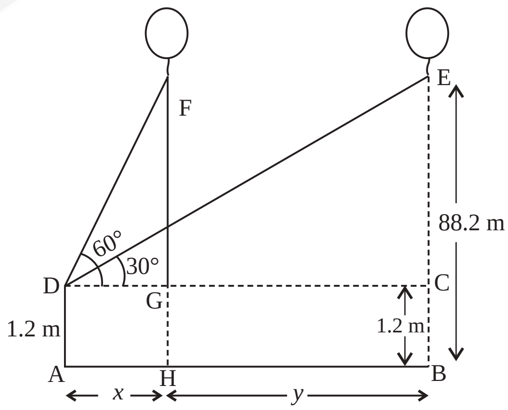
Hence, the distance between the two ships is $75(\sqrt{3} - 1)$ m.

14. **1.2m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.**



Sol. : Let AB be the horizontal line on which a girl is standing whose length $AD = 1.2$ m.

Let, $FH = EB = 88.2$ m is the height of the balloon from the horizontal line AB. The angle of elevation of the balloon from the girl's eye D are 60° and 30°.



i.e., $\quad \angle FDC = 60°$

and $\quad \angle EDC = 30°$

$\therefore \quad CE = FG$

$= 88.2 - 1.2 = 87$ m

Let, $AH = DG = x$ m

and $HB = GC = y$ m

In right angle ΔFGD,

$$\tan 60° = \frac{FG}{DG}$$

$$\Rightarrow \quad \sqrt{3} = \frac{87}{x}$$

$$\therefore \quad x = \frac{87}{\sqrt{3}} \quad \text{...(i)}$$

In right angle ΔECD,

$$\tan 30° = \frac{EC}{DC} = \frac{EC}{DG + GC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{87}{x + y}$$

$$\Rightarrow \quad x + y = 87\sqrt{3}$$

$$\Rightarrow \quad \frac{87}{\sqrt{3}} + y = 87\sqrt{3}$$

$$\Rightarrow \quad y = 87\sqrt{3} - \frac{87}{\sqrt{3}}$$

$$= 87\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$$

$$= 87\left(\frac{3-1}{\sqrt{3}}\right) = \frac{87 \times 2}{\sqrt{3}}$$

$$= \frac{87 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{87 \times 2 \times \sqrt{3}}{3}$$

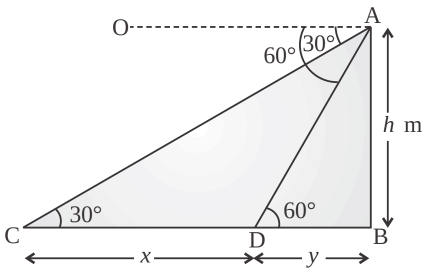
$$\therefore \quad y = 29 \times 2 \times \sqrt{3}$$

$$= 58\sqrt{3} \text{ m}$$

Hence, the distance covered by the balloon $= 58\sqrt{3}$ m.

15. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

Sol. : Let *AB* be a tower whose height is *h* metres. A boy standing at the top of a tower observes a car at an angle of depression of 30°, which is approaching the height of the tower with a uniform speed. After 6 seconds the angle of depression of the car becomes 60°.



$\angle OAC = 30°, \angle OAD = 60°$

Now, $\angle OAC = \angle ACB = 30°$

[Alternate interior angle]

$\angle OAD = \angle ADB = 60°$

[Alternate interior angle]

Let, $CD = x$ and $BD = y$

In right angle ΔABD,

$$\tan 60° = \frac{AB}{BD}$$

$$\Rightarrow \sqrt{3} = \frac{h}{y}$$

$$\Rightarrow h = \sqrt{3}y \quad \text{...(i)}$$

In right angle ΔABC,

$$\tan 30° = \frac{AB}{BC} = \frac{AB}{BD + DC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{y + x}$$

$$\Rightarrow y + x = h\sqrt{3} \quad \text{...(ii)}$$

Putting the value of h from. eqn. (i) into eqn. (ii) we get,

$$y + x = \sqrt{3}y \times \sqrt{3}$$

$$\Rightarrow y + x = 3y$$

$$\Rightarrow x = 3y - y$$

$$x = 2y \Rightarrow y = \frac{x}{2}$$

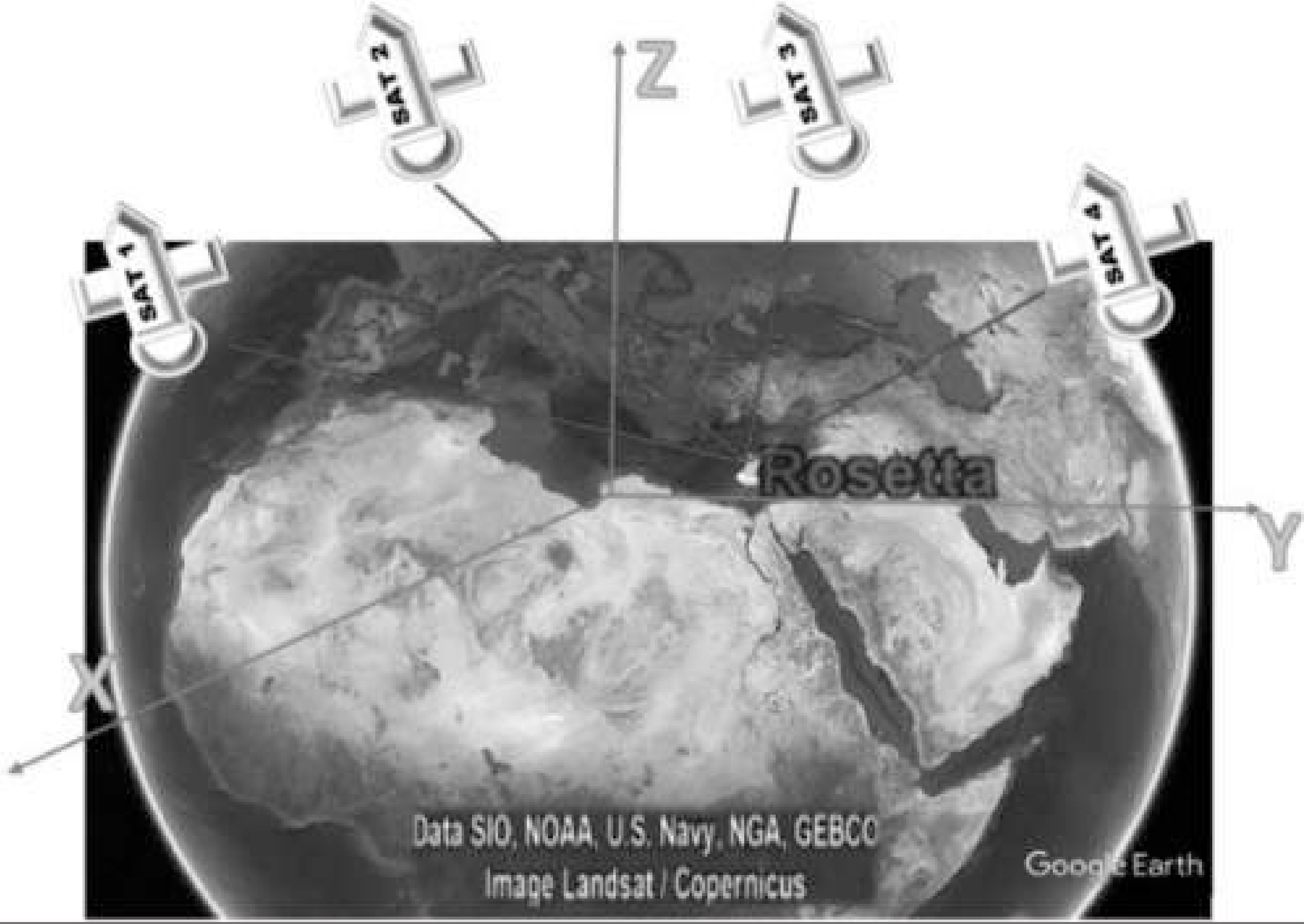
$\because$ CD = Time taken to cover a distance of x m = 6 sec.

$\therefore$ y = Time taken to cover $\frac{x}{2}$ m distance = $\frac{6}{2}$ = 3 sec.

Hence, the car will take 3 seconds to reach the tower.

Circles 10

NCERT SOLUTIONS



What's inside

– *In-Chapter Q's (solved)*

– *Textbook Exercise Q's (solved)*

Exercise – 10.1

1. How many tangents can a circle have ?

Sol. : There can be infinitely many tangents to a circle.

2. Fill in the blanks :

(i) A tangent to a circle intersects it in .... point(s).

(ii) A line intersecting a circle in two points is called a ....

(iii) A circle can have .... parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called ....

Sol. : (i) One [A tangent to a circle intersects it in one point.]

(ii) Secant

If a line intersects a circle at two distinct points, then this line is called a secant of the circle.

(iii) Two [A circle can have two parallel tangents at the most].

(iv) Point of Contact

A line meeting a circle at exactly one point is called a tangent to the circle. And the point at which the line touches the circle is called the point of contact.

3. A tangent *PQ* at a point *P* of a circle of radius 5 cm meets a line through the centre *O* at a point *Q* so that *OQ* = 12 cm, Length *PQ* is :

(a) 12 cm (b) 13 cm

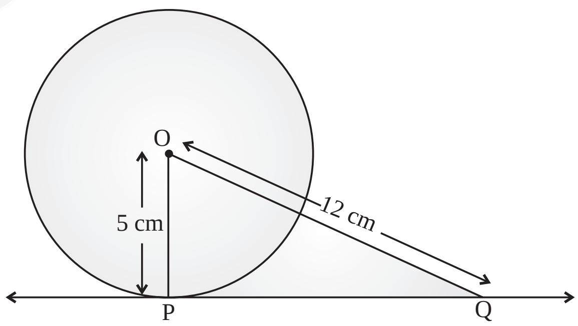
(c) 8·5 cm (d) $\sqrt{119}$ cm

Sol. : (d) Given, *PQ* is the tangent which touches the circle at the point *P*. *OP* = 5 cm is the radius of the circle. *OQ* = 12 cm.

$\because$ *OP* is the radius of the circle, so the tangent will be perpendicular to *PQ*.

$\therefore$ In right angle ΔOPQ,

$$(OQ)^2 = (OP)^2 + (PQ)^2$$ [By Pythagoras theorem]



$\Rightarrow \quad 12^2 = 5^2 + PQ^2$

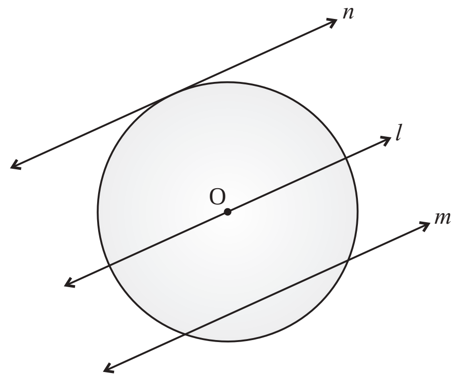
$\Rightarrow \quad 144 - 25 = PQ^2$

$\Rightarrow \quad 119 = PQ^2$

$\therefore \quad PQ = \sqrt{119}$ cm

4. Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

Sol. : First draw a circle (O, r) then draw a line l. Now draw a line m parallel to l which is a secant line and draw a line n parallel to l which is a tangent to the circle.



Exercise – 10.2

In Q. 1 to 3, choose the correct option and give justification.

1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is :

(a) 7 cm (b) 12 cm

(c) 15 cm (d) 24·5 cm

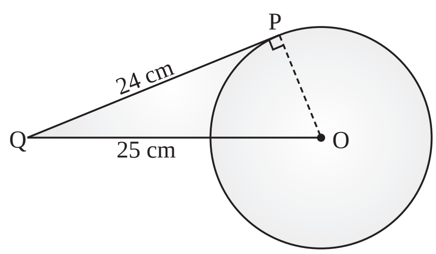
Sol. :(a) Let, P be a point on the circle, then the length of the tangent $QP = 24$ cm. And $QO = 25$ cm. Now join OP.

We know that the radius OP is perpendicular to the tangent PQ.

In right angle ΔOPQ,

$$OQ^2 = OP^2 + PQ^2 \quad \text{[By Pythagoras theorem]}$$

$$\Rightarrow \quad 25^2 = OP^2 + 24^2$$



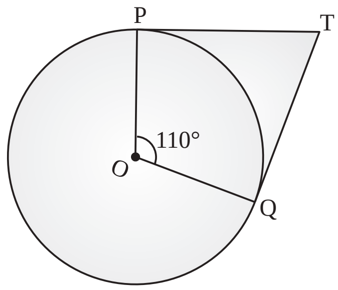
$$\Rightarrow \quad 625 = OP^2 + 576$$

$$\Rightarrow \quad OP^2 = 625 - 576 = 49$$

$$\therefore \quad OP = \sqrt{49} = 7 \text{ cm}$$

Hence, the radius of the circle is 7 cm.

2. In figure below, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110°$, then $\angle PTQ$ is equal to :



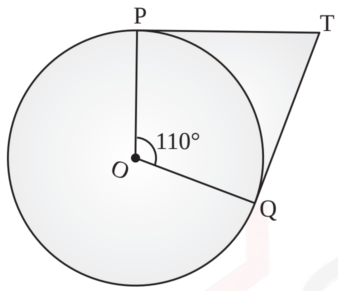
(a) 60º (b) 70º

(c) 80º (d) 90º

Sol. : (b) Given, *TP* and *TQ* are two tangents to the circles

$\because$ $OP \perp PT$ and $OQ \perp TQ$ [The radius of the circle is perpendicular]

$\therefore$ $\angle OPT = 90^\circ$ and $\angle OQT = 90^\circ$



In quadrilateral *OPTQ*,

$\angle POQ + \angle OQT + \angle PTQ + \angle OPT = 360^\circ$

$110^\circ + 90^\circ + \angle PTQ + 90^\circ = 360^\circ$

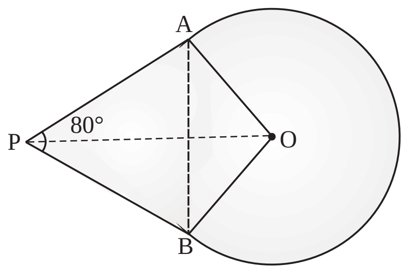
$\angle PTQ = 360^\circ - 290^\circ = 70^\circ$

3. If tangents *PA* and *PB* from a point *P* to a circle with centre *O* are inclined to each other at angle of 80°, then ∠*POA* is equal to :

(a) 50º (b) 60º

(c) 70º (d) 80º

Sol. : (a) We know that the radius of a circle is perpendicular to the tangent at the tangent point.



$\therefore$ $OA \perp PA$ and $OB \perp PB$

Thus, $\angle OAP = 90^\circ$ and $\angle OBP = 90^\circ$

In quadrilateral *OAPB*,

$\angle APB + \angle OBP + \angle OAP + \angle AOB = 360^\circ$

$80^\circ + 90^\circ + 90^\circ + \angle AOB = 360^\circ$

$\angle AOB = 360° - 260° = 100°$

Now, in ΔOAP and ΔOBP,

$AP = BP$ [$\therefore$ Tangent line to a circle are equal]

$OP = OP$ [Common side]

$OA = OB$ [Radii of circle]

$\therefore$ $\Delta OAP \cong \Delta OBP$ [By *SSS* Congruency]

Hence, $\angle POA = \angle POB$ [By *CPCT*] ... (i) Thus, *OP* is the bisector of $\angle AOB$.

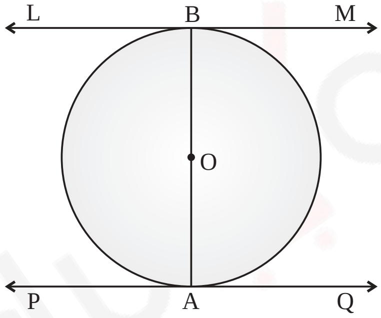
$\therefore$ $\angle POA = \frac{1}{2} \angle AOB$

$= \frac{1}{2} \times 100°$

$= 50°$

4. **Prove that the tangents drawn at the ends of a diameter of a circle are parallel.**

Sol. : Let *LM* and *PQ* be the tangents drawn to the circle at points *B* and *A* respectively.



We know that the radius of a circle is perpendicular to the tangent at the tangent point.

$\therefore$ $AB \perp LM$ and $AB \perp PQ$

Thus, $\angle ABL = 90°$ and $\angle BAQ = 90°$

Then, $\angle ABL = \angle BAQ$

But these are alternate interior angles.

So, $LM \parallel PQ$

Hence, the tangents drawn at the ends of the diameter of the circle are parallel to each other.

Hence Proved

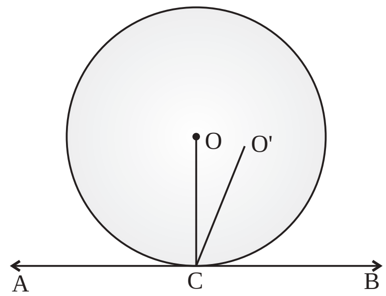
5. **Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.**

Sol. : Given : *AB* is a tangent to a circle with centre *O* at a point *C*.

To Prove : The perpendicular drawn from the tangent point *C* to the tangent *AB* passes through the centre *O* of the circle.

Construction : Join *OC* and *O'C*.

Proof : Let the perpendicular passes through another point *O'*.



$\because$ A tangent drawn at a point to a circle is perpendicular to the radius passing through the point of contact.

$\therefore \quad OC \perp AB \Rightarrow \angle OCB = 90°$

and $\quad O'CB = 90°$ [Let $O'C \perp AB$]

$\therefore \quad \angle OCB = \angle O'CB$

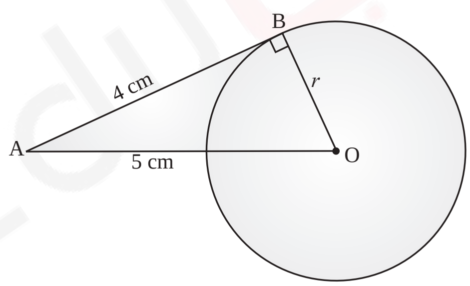
which is possible only when O and O' are coincide, so the perpendicular drawn from the point of contact C to the tangent AB passes through the centre O.

Hence Proved

6. The length of a tangent from a point *A* at a distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Sol. : Let, there be a point A at a distance of 5 cm outside the circle (O, r).

i.e., $\quad OA = 5$ cm



and length of tangent $AB = 4$ cm

Let r is the radius of the circle.

We know that the radius of a circle is perpendicular to the tangent at a tangent point.

So, in right angle ΔABO,

$$(AO)^2 = (OB)^2 + (AB)^2$$

$$\Rightarrow \quad 5^2 = r^2 + 4^2$$

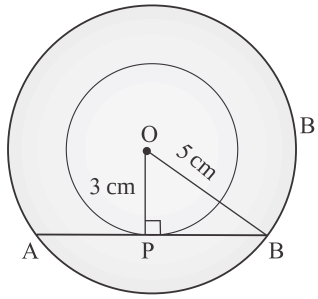
$$\Rightarrow \quad 25 = r^2 + 16$$

$$\Rightarrow \quad r^2 = 25 - 16 = 9$$

$$\therefore \quad r = 3 \text{ cm}$$

7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Sol. : Let C_1 and C_2 be two concentric circles of radii 3 cm and 5 cm respectively. *i.e.*, $OP = 3$ cm, $OB = 5$ cm



Let the chord AB of the larger circle C touches the smaller circle C at the point P.

Now from the centre O, draw a perpendicular to AB which meets AB at the mid point P.

$\therefore \quad AP = BP$

In right angled OPB,

$$OB^2 = OP^2 + PB^2$$

$\Rightarrow \quad 5^2 = 3^2 + PB^2$

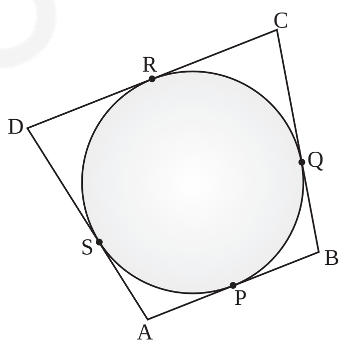
$\Rightarrow \quad 25 - 9 = PB^2$

$\Rightarrow \quad PB^2 = 16$

$\therefore \quad PB = 4$ cm

$\therefore$ Length of chord $AB = 2PB = 2 \times 4 = 8$ cm

8. A quadrilateral *ABCD* is drawn to circumscribe a circle. Prove that : *AB* + *CD* = *AD* + *BC*



Sol. : Given : Quadrilateral *ABCD* circumscribing a circle.

To Prove : $AB + CD = AD + BC$.

Proof : We know that tangents drawn from an external point to a circle are of equal length.

The external point is A, then

$AP = AS$ ...(i)

The external point is B, then

$PB = BQ$ ...(ii)

The external point is C, then

$$CR = CQ \quad \text{...(iii)}$$

The external point is D, then

$$DR = DS \quad \text{...(iv)}$$

Adding eqn. (i), (ii), (iii) and (iv), we get

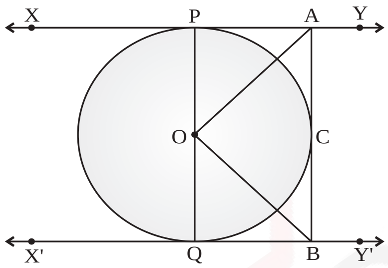
$$(AP + PB) + (CR + DR) = AS + BQ + CQ + DS$$

$$\Rightarrow \quad AB + CD = (AS + DS) + BQ + CQ$$

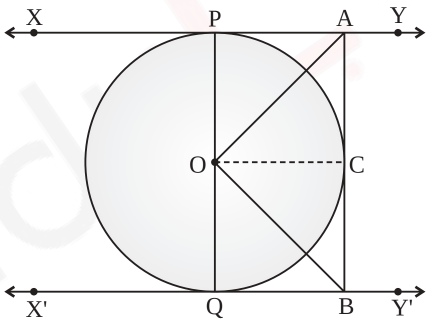
$$\therefore \quad AB + CD = AD + BC$$

Hence Proved

9. In figure below, XY and $X'Y'$ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and $X'Y'$ at B. Prove that $\angle AOB = 90°$.



Sol. : Given : XY and $X'Y'$ are two parallel tangents. Another tangent AB touches the circle at C and intersects XY at A and $X'Y'$ at B.



To Prove : $\angle AOB = 90°$

Construction : Join OC.

Proof : We know that tangents drawn from an external point to a circle are of equal lengths.

$$AP = AC \quad \text{...(i)}$$

In ΔAPO and ΔACO,

$AP = AC$

$AO = AO$ [Common]

$OP = OC$ [Radii of circle]

$\Delta APO \cong \Delta ACO$ [By SSS Congruency]

and $\angle PAO = \angle CAO$ (By $CPCT$)

So, $\angle PAC = 2\angle CAO$ ...(ii)

Similarly, we can prove that

$\angle CBO = \angle OBQ$

$\angle CBQ = 2\angle CBO$ ...(iii)

$\because$ $XY \parallel X'Y'$

$\therefore \angle PAC + \angle QBC = 180^\circ$ [The sum of interior angles on the same side of a transversal is 180°.]

$\Rightarrow 2\angle CAO + 2\angle CBO = 180^\circ$

[From eqn. (i) and (ii)]

$\Rightarrow \angle CAO + \angle CBO = \frac{180^\circ}{2} = 90^\circ$ ...(iv)

In ΔAOB,

$\angle BAO + \angle ABO + \angle AOB = 180^\circ$

$\angle CAO + \angle CBO = 180^\circ - \angle AOB$

$\therefore \angle BAO = \angle CAO$ and $\angle ABO = \angle CBO$

$180^\circ - \angle AOB = 90^\circ$ [From eqn. (iv)]

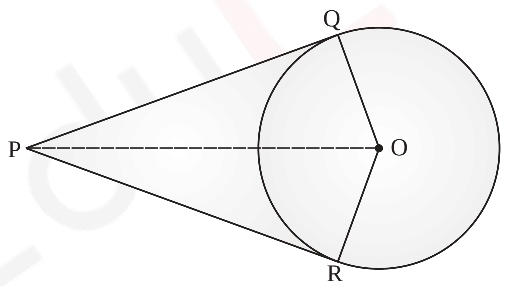
$\therefore$ $\angle AOB = 180^\circ - 90^\circ = 90^\circ$

10. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Sol. : Given : Two tangents PQ and PR are drawn from an external points P to a circle with centre O.

To Prove : $\angle QOR = 180^\circ - \angle QPR$

or $\angle QOR + \angle QPR = 180^\circ$



Proof : We know that the radius of a circle is perpendicular to the tangent line at the point of contact.

$\therefore$ $OQ \perp PQ$ and $OR \perp PR$

i.e., $\angle OQP = 90^\circ$ and $\angle ORP = 90^\circ$

In quadrilateral $PQOR$,

$\angle OQP + \angle QPR + \angle ORP + \angle QOR = 360^\circ$

$90^\circ + \angle QPR + 90^\circ - \angle QOR = 360^\circ$

$\angle QPR + \angle QOR = 360^\circ - 180^\circ$

$\therefore$ $\angle QOR + \angle QPR = 180^\circ$

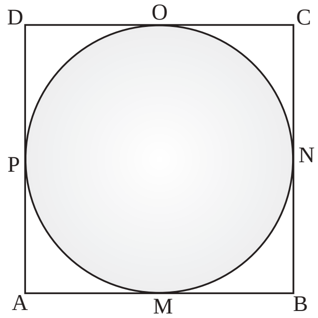
Hence Proved

11. Prove that the parallelogram circumscribing a circle is a rhombus.

Sol. : Given : Let parallelogram $ABCD$ is circumscribed on a circle.

To Prove : $ABCD$ is a rhombus.

i.e., $AB = BC = CD = DA$



Proof : We know that tangents drawn from an external point to a circle are equal in length.

$\therefore \quad AM = AP \quad$ ...(i)

$BM = BN \quad$ ...(ii)

$\therefore \quad CO = CN \quad$ ...(iii)

$\therefore \quad DO = DP \quad$ ...(iv)

Adding all the equations,

$(AM + BM) + (CO + DO)$

$= AP + BN + CN + DP$

$\Rightarrow \quad AB + CD = (AP + PD) + (BN + NC)$

$\Rightarrow \quad AB + CD = AD + BC \quad$ ...(i)

$\because$ *ABCD* is a parallelogram.

$\therefore \quad AB = CD$ and $BC = AD \quad$ ...(ii)

From eqn. (i),

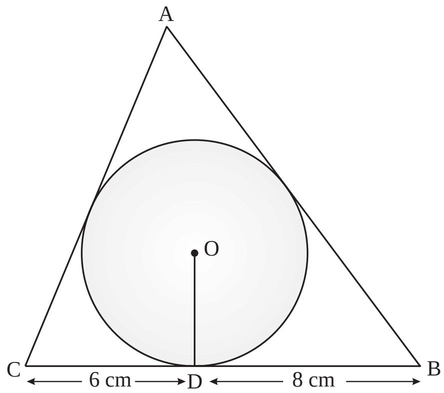
$2AB = 2BC \Rightarrow AB = BC \quad$ ...(iii)

From eqn. (ii) and (iii),

$AB = BC = CD = DA$

Thus, *ABCD* is a rhombus.

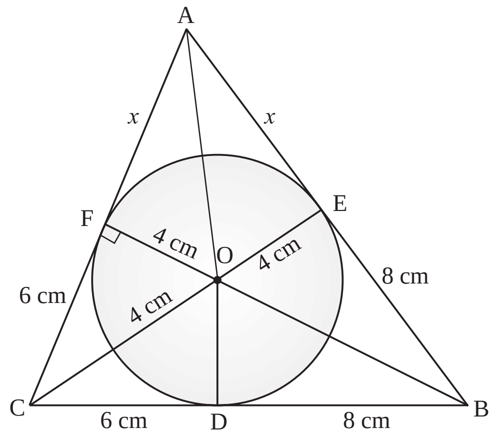
12. A triangle *ABC* is drawn to circumscribe a circle of radius 4 cm such that the segments *BD* and *DC* into which *BC* is divided by the point of contact *D* are of lengths 8 cm and 6 cm respectively. Find the sides *AB* and *AC*.



Sol. : Given : $CD = 6$ cm, $BD = 8$ cm and radius = 4 cmJoin *OC*, *OA* and *OB*.

We know that the lengths of tangents are drawn from an external point are equal.

$\therefore \; CD = CF = 6$ cm and $BD = BE = 8$ cm



Let $AF = AE = x$ cm

In ΔOCB,

Area of triangle $A_1 = \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times CB \times OD$$

$$= \frac{1}{2} \times 14 \times 4 = 28 \text{ cm}^2$$

In ΔOCA,

Area of triangle $A_2 = \frac{1}{2} \times AC \times OF$

$$= \frac{1}{2}(6 + x) \times 4$$

$$= 12 + 2x$$

In ΔOBA,

Area of triangle $A_3 = \frac{1}{2} \times AB \times OE$

$$= \frac{1}{2}(8 + x) \times 4$$

$$= 16 + 2x$$

Now, Semiperimeter of ΔABC

$$s = \frac{1}{2}(AB + BC + CA)$$

$$s = \frac{1}{2}(x + 8 + 14 + 6 + x)$$

$$s = 14 + x$$

Now, Area of ΔABC

$$= A_1 + A_2 + A_3$$

$$= 28 + (12 + 2x) + (16 + 2x)$$

$$= 56 + 4x \qquad \text{...(i)}$$

Using Heron's formula,

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{(14 + x)(14 + x - 14)(14 + x - x - 6)(14 + x - x - 8)}$$

$$= \sqrt{(14 + x) \times x \times 8 \times 6}$$

$= \sqrt{(14+x)x \times 48}$ ...(ii)

From eqn. (i) and (ii),

$\sqrt{(14+x)x \times 48} = 56 + 4x = 4(14 + x)$

Squaring both sides,

$(14 + x)48x = 4^2(14 + x)^2$

$\Rightarrow \quad 3x = 14 + x$

$\Rightarrow \quad 2x = 14 \Rightarrow x = 7$

$\therefore$ Length of $AC = 6 + x = 6 + 7 = 13$ cm

andLength of $AB = 8 + x = 8 + 7 = 15$ cm

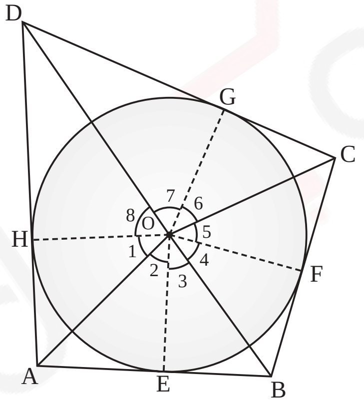
13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Sol. : Let *ABCD* be a quadrilateral circumscribing a circle. The circle touches the quadrilateral at point *E, F, G* and *H*.

To Prove : $\angle AOB + \angle COD = 180^\circ$

and $\angle AOD + \angle BOC = 180^\circ$

Construction : Join *OH*, *OE*, *OF* and *OG*.



Proof : In ΔAOE and ΔAOH,

$AO = AO$ [Common side]

$OE = OH$ [Radii of circle]

$AE = AH$ [Tangents]

$\therefore \quad \Delta AOE \cong \Delta AOH$ [By *SSS* Congruency]

So, $\angle AOE = \angle AOH$ [By *CPCT*]

or $\angle 2 = \angle 1$ ...(i)

Similarly, ΔEOB and ΔFOB,

$\angle 3 = \angle 4$ ...(ii)

Similarly, ΔCOF and ΔCOG,

$\angle 5 = \angle 6$ ...(iii)

Similarly, ΔDOG and ΔDOH,

$\angle 7 = \angle 8$ ...(iv)

$\because \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$

$\angle 2 + \angle 2 + \angle 3 + \angle 3 + \angle 6 + \angle 6 + \angle 7 + \angle 7 = 360^\circ$

$2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360^\circ$

$(\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180^\circ$

$\angle AOB + \angle COD = 180^\circ$

Again, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$

$\angle 1 + \angle 1 + \angle 4 + \angle 4 + \angle 5 + \angle 5 + \angle 8 + \angle 8 = 360^\circ$

$2(\angle 1 + \angle 4 + \angle 5 + \angle 8) = 360^\circ$

$(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180^\circ$

$\angle AOD + \angle BOC = 180^\circ$

Hence Proved

Area Related to Circles 11

NCERT SOLUTIONS

What's inside

– *In-Chapter Q's (solved)*
– *Textbook Exercise Q's (solved)*

Exercise – 11.1

Unless stated otherwise, use $\pi = \frac{22}{7}$.

1. **Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60º.**

Sol. : Given,

Radius of circle = 6 cm, and θ = 60°

We know that,

$$\text{Area of sector} = \pi r^2 \times \frac{\theta}{360}$$

$$= \frac{22}{7} \times 6 \times 6 \times \frac{60}{360}$$

$$= \frac{22}{7} \times 6 = \frac{132}{7} \text{ cm}^2$$

2. **Find the area of a quadrant of a circle whose circumference is 22 cm.**

Sol. : Circumference of Circle= 22 cm

$$2\pi r = 22$$

$$2 \times \frac{22}{7} \times r = 22$$

$$r = \frac{7}{2} \text{cm}$$

$$\therefore \text{Area of a quadrant of circle} = \frac{\pi r^2}{4}$$

$$= \frac{22}{7} \times \frac{1}{4} \times \frac{7}{2} \times \frac{7}{2}$$

$$= \frac{77}{8} \text{cm}^2$$

3. **The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.**

Sol. : Length of the minute hand = 14 cm

Hence, the minute hand will make a sector of radius 14 cm

$\therefore$ Angle made in 60 min. = 360°

$\therefore$ Angle made in 1 min. $= \frac{360°}{60°} = 6°$

$\therefore$ Angle made in 5 min. = 6° × 5° = 30°

$$\text{Hence, Required area} = \pi r^2 \times \frac{\theta}{360°}$$

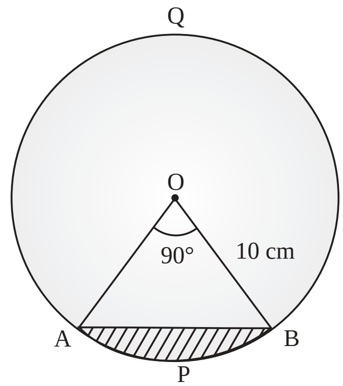
$$= \frac{22}{7} \times 14 \times 14 \times \frac{30}{360°}$$

$$= \frac{154}{3} \text{cm}^2$$

4. **A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding : (i) minor segment (ii) major sector. (Use π = 3.14)**

Sol. : Given, $\angle AOB = 90°$

andRadius of circle r = 10 cm



(i) Area of minor segment = Area of sector *OAPB* – Area of Δ*OAB*

Now area of sector *OAPB*

$$= \pi r^2 \frac{\theta}{360°} = \frac{22}{7} \times 10 \times 10 \times \frac{90°}{360°}$$

$$= \frac{2200}{28} = \frac{550}{7} \text{ cm}^2$$

andArea of Δ*OAB* $= \frac{1}{2} \times$ base × height

$$= \frac{1}{2} \times OA \times OB$$

$$= \frac{1}{2} \times 10 \times 10$$

$$= 50 \text{ cm}^2$$

∴ Area of minor segment

$$= \frac{550}{7} - 50$$

$$= \frac{550 - 350}{7} = \frac{200}{7}$$

$$= 28.5 \text{ cm}^2$$

(ii) Area of major segment

= Area of Circle – Area of *OABP*

$$= \frac{22}{7} \times 10 \times 10 - \frac{550}{7}$$

$$= \frac{2200 - 550}{7} = \frac{1650}{7}$$

$$= 235.5 \text{ cm}^2$$

5. In a circle of radius 21 cm an arc subtends an angle of 60° at the centre. Find :

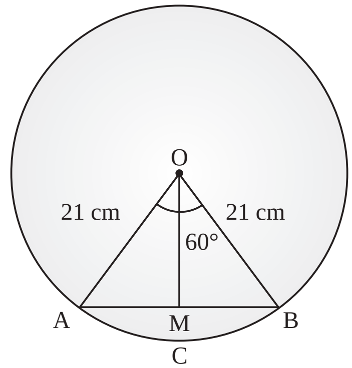
(i) the length of the arc

(ii) area of the sector formed by the arc.

(iii)area of the segment formed by the corresponding chord.

Sol. : ∵ The arc *ACB* subtends an angle of 60° at the centre of the circle.

∴ $\theta = 60°$ and Radius = 21 cm



(i) Length of arc $ACB = 2\pi r \times \frac{\theta}{360°}$

$= 2 \times \frac{22}{7} \times 21 \times \frac{60}{360°}$

$= \frac{2 \times 22 \times 3 \times 1}{6} = 22 \text{ cm}$

(ii) Area of the sector formed by the arc

$= \pi r^2 \times \frac{\theta}{360°}$

$= \frac{22}{7} \times 21 \times 21 \times \frac{60}{360°}$

$= \frac{22 \times 3 \times 21}{6} = 231 \text{ cm}^2$

(iii) From the centre of the circle, a perpendicular is drawn on AB

or $OM \perp AB$

$\therefore$ $AM = MC$

and $\angle AOD = \angle BOC = 30°$

In right angled ΔAOM,

$\sin 30° = \frac{AM}{AO}$

$\frac{1}{2} = \frac{AM}{21}$

$AM = \frac{21}{2}$

$\therefore$ $AB = 2AM = 21 \text{ cm}$

and $\cos 30° = \frac{OM}{OA}$

$\Rightarrow$ $\frac{\sqrt{3}}{2} = \frac{OM}{21}$

$\Rightarrow$ $OM = \frac{21\sqrt{3}}{2}$

$\therefore$ Area of $\Delta OAB = \frac{1}{2} \times AB \times OM$

$= \frac{1}{2} \times 21 \times 21 \frac{\sqrt{3}}{2}$

$= \frac{441\sqrt{3}}{4} \text{ cm}^2$

$\therefore$ Area of segment = Area of sector $OACB$ – Area of ΔOAB

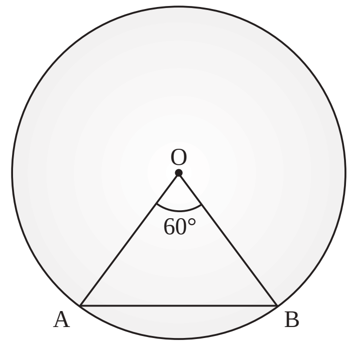
$= 231 - \frac{441\sqrt{3}}{4} \text{ cm}^2$

6. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (use π = 3.14 and $\sqrt{3}$ = 1.732)

Sol. : Radius of circle = 15 cm

Chord AB of the circle subtends an angle of 60° at the centre.

Here, $OA = OB = 15$ cm



Area of minor segment

$$= \pi r^2 - \frac{\theta}{360°} - \frac{1}{2} r^2 \sin\theta$$

$$= 3.14 \times (15)^2 \times \frac{120°}{360°} - \frac{1}{2} \times (15)^2 \times \sin 120°$$

$$= 3.14 \times 225 \times \frac{1}{6} - \frac{1}{2} \times 225 \times \frac{\sqrt{3}}{2}$$

$$= 3.14 \times 37.5 - \frac{1}{2} \times 225 \times \frac{1.732}{2}$$

= 117.75 – 97.425

= 20.325 cm^2

Area of major segment

= Area of Circle – Area of minor segment

= πr^2 – 20.325

= 3.15 × $(15)^2$ – 20.325

= 706.5 – 20.325

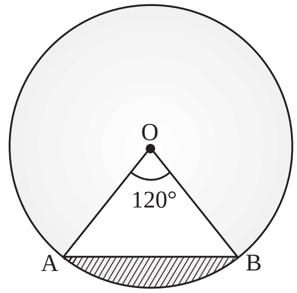
= 686.175 cm^2

7. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use π = 3.14 and $\sqrt{3}$ = 1.73)

Sol. : Given, Radius of circle = 12 cm

Angle subtended by the chord at the centre = 120°

From the centre of the circle, a perpendicular *OM* is drawn on the chord *AB i.e., OM* ⊥ *AB*.



Area of minor segment

$$= \pi r^2 . \frac{\theta}{360°} - \frac{1}{2} r^2 \sin\theta$$

$$= 3.14 \times (12)^2 \times \frac{12}{360°} \quad - \frac{1}{2} \times (12)^2 \sin 120°$$

$$= 3.14 \times 144 \times \frac{1}{3} \quad - \frac{1}{2} \times 144 \times \sin(180° - 60°)$$

$$= 3.14 \times 48 - 72 \times \sin 60°$$

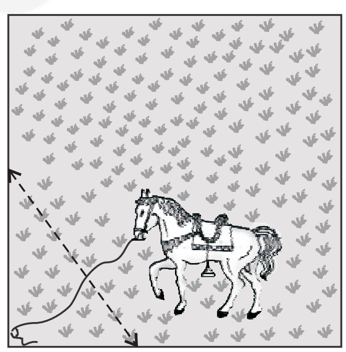
$$= 150.72 - 72 \times \frac{\sqrt{3}}{2}$$

$$= 150.72 - 72 \times \frac{1.73}{2}$$

$$= 150.72 - 62.28$$

$$= 88.44 \text{ cm}^2$$

8. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find :



(i) the area of that part of the field in which the horse can graze.

(ii)the increase in the grazing area if the rope were 10m long instead of 5m (Use π = 3.14)

Sol. : Given, Side of square = 15 m

∴ Area of square = $(\text{Side})^2$

$= 15^2 = 225 \text{ m}^2$

Length of rope = 5 m

∴ Radius of arc = 5 m

(i) Area grazed by horse

A_1 = Area of sector

$$= \pi r^2 \times \frac{\theta}{360°} = \frac{22}{7} \times 5^2 \times \frac{90°}{360°}$$

$$= \frac{275}{14} = 19.64 \text{ cm}^2$$

(ii) If length of rope = 10 m

∴ Area grazed by horse A_2

$$= \pi r^2 \times \frac{\theta}{360°}$$

$$= \frac{22}{7} \times 10^2 \times \frac{90°}{360°}$$

$$= \frac{22 \times 25}{7} = 78.57 \text{ cm}^2$$

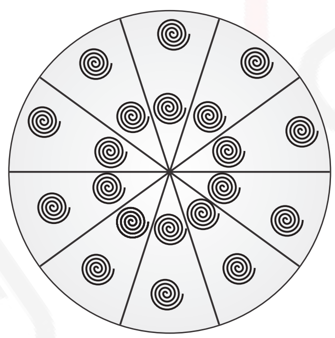
∴ Increase in the grazing area

$= A_2 - A_1$

$= 78.57 - 19.64$

$= 58.93 \text{ cm}^2$

9. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in figure below. Find :



(i) the total length of the silver wire required.

(ii)the area of each sector of the brooch.

Sol. : Given, diameter of circular brooch = 35 mm

∴ Radius $= \frac{35}{2}$ mm

Length of 5 diameter = 5 × 35

= 175 mm

Circumference of Circle

$= 2\pi r = 2 \times \frac{22}{7} \times \frac{35}{2}$

= 110 mm

(i) Required length of wire of silver = Length of 5 diameter + circumference of Circle

= 175 + 110 = 285 mm

(ii) Here, the circle is divided into 10 equal parts

Hence, angle of each sector $= \frac{360°}{10} = 36°$

∵ Area of each sector of brooch

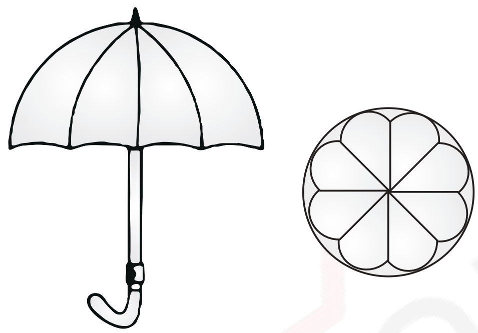
$$= \pi r^2 \times \frac{\theta}{360°}$$

$$= \frac{22}{7}\times\left(\frac{35}{2}\right)^2\times\frac{36}{360°}$$

$$= \frac{22}{7}\times\frac{35}{2}\times\frac{35}{2}\times\frac{1}{10}$$

$$= \frac{385}{4}\text{mm}^2$$

10. An umbrella has 8 ribs which are equally spaced. Assuming umbrella to be a flat circle of radius 45 cm, find area between the two consecutive ribs of the umbrella.



Sol. : ∵ An umbrella has 8 ribs

The central angle of the umbrella is 360°.

∴ Angle between two ribs $= \frac{360°}{8} = 45°$

Radius = 45 cm

∴ Area between two consecutive ribs

= Area of one sector of umbrella

$$= \pi r^2 \times \frac{\theta}{360°} = \frac{22}{7}\times 45°\times 45°\times\frac{45°}{360°}$$

$$= \frac{22\times 45\times 45}{7\times 8}$$

$$= \frac{11\times 45\times 45}{28} = \frac{22275}{28}\text{ cm}^2$$

11. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Sol. : Blade of wiper length (r) = 25 cm

and Angle made by blade θ = 115°

∴ Area cleaned by blade

= Area of sector made by blade

$$= \pi r^2 \times \frac{\theta}{360°}$$

$$= \frac{22}{7} \times 25 \times 25 \times \frac{115°}{360°}$$

$$= \frac{22 \times 625 \times 23}{7 \times 72}$$

$$= \frac{158125}{252} \text{ cm}^2$$

$\therefore$ Area cleaned by both the blades

$$= 2 \times \frac{158125}{252} = \frac{158125}{126} \text{cm}^2$$

12. **To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use π = 3.14)**

Sol. : Given,Sector of angle = 80°

and Radius = 16.5 km

$\therefore$ Area of sector $= \pi r^2 \frac{\theta}{360°}$

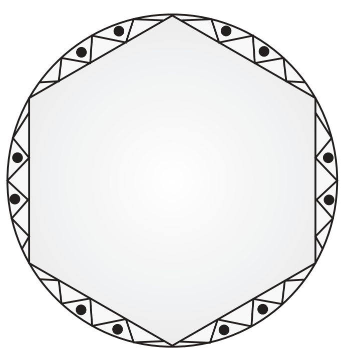
$$= \frac{3.14 \times (16.5)^2 \times 80°}{360°}$$

$$= \frac{2 \times 3.14 \times 272.25}{9}$$

$$= \frac{1709.73}{9} = 189.97 \text{ cm}^2$$

which is the desired area in which ships can be warned.

13. **A round table cover has six equal designs as shown in figure below. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹ 0.35 per cm^2 . (Use $\sqrt{3} = 1.7$)**



Sol. : Central angle of circle = 360°

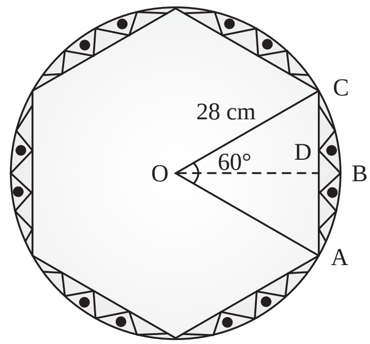
$\therefore$ Angle of each sector $= \frac{360°}{6} = 60°$

Now, we find the area of a sector of the circle.

Here, AC is chord of circle. Draw $OD \perp AC$ which bisects the chord AC.

$$AD = DC$$

$\therefore$ $\angle COD = \angle AOD = 30°$



In right angled ΔOCD,

$$\sin 30° = \frac{DC}{OC}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{DC}{28}$$

$$\Rightarrow \quad DC = \frac{28}{2} = 14 \text{ cm}$$

and $$\cos 30° = \frac{OD}{OC}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{OD}{28}$$

$$\Rightarrow \quad OD = \frac{28\sqrt{3}}{2} = 14\sqrt{3} \text{ cm}$$

$$= 14 \times 1.7 = 23.8 \text{ cm}$$

$$AC = 2 \times CD = 2 \times 14$$

$$= 28 \text{ cm}$$

Area of $\Delta AOC = \frac{1}{2} \times AC \times OD$

$$= \frac{1}{2} \times 28 \times 23.8 = \frac{666.4}{2}$$

$$= 333.2 \text{ cm}^2$$

Now area of sector *OABCO*

$$= \pi r^2 \times \frac{\theta}{360°}$$

$$= \frac{22}{7} \times 28^2 \times \frac{60°}{360°}$$

$$= \frac{22 \times 28 \times 28}{7 \times 6}$$

$$= \frac{11 \times 4 \times 28}{3}$$

$$= \frac{1232}{3} = 410.67 \text{ cm}^2$$

$\therefore$ Area of segment *ABCA* = Area of sector *OABCO* – Area of ΔAOC

$$= 410.67 - 333.2$$

$$= 77.47 \text{ cm}^2$$

$\therefore$ Area of six segments

$$= 6 \times 77.47$$

$$= 464.82 \text{ cm}^2$$

Hence, the cost of making the designs at the rate of ₹ 0.35 per cm^2 = 464.82 × 0.35

= ₹ 162.68

14. Tick the correct answer in the following :
Area of a sector of angle *P* (in degrees) of a circle with radius R is :

(a) $\frac{P}{180} \times 2\pi r$ (b) $\frac{P}{180} \times \pi R^2$

(c) $\frac{P}{360} \times \pi R$ (d) $\frac{P}{720} \times 2\pi R^2$

Sol. : (d) Given : $q = P°, r = R$

$\therefore$ Area of sector $= \frac{\pi r^2 \theta}{360°}$

$= \pi R^2 \times \frac{P°}{360°}$

$= 2\pi R^2 \times \frac{P°}{720°}$

Surface Areas and Volumes 12

NCERT SOLUTIONS

What's inside

– *In-Chapter Q's (solved)*

– *Textbook Exercise Q's (solved)*

Exercise – 12.1

Unless stated otherwise, take $\pi = \frac{22}{7}$.

1. **2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.**

Sol. : Given,

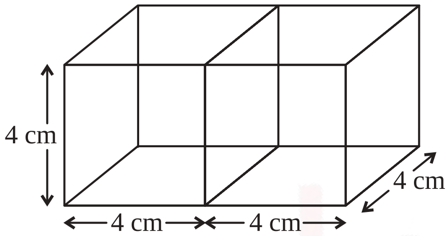
Volume of a Cube = 64 cm^3

Let, the length of the side of a cube is 4*a* cm then

$a^3 = 64$

$\Rightarrow a^3 = (4)^3$

$\Rightarrow a = 4$ cm



When we join 2 cubes end to end we get a cuboid whose

Length $(l) = 4 + 4 = 8$ cm,

Breadth $(b) = 4$ cm

andHeight $(h) = 4$ cm

$\therefore$ Surface area of the cuboid

$= 2(lb + bh + hl)$

$= 2(8 \times 4 + 4 \times 4 + 4 \times 8)$

$= 2(32 + 16 + 32)$

$= 2 \times 80 = 160$ cm^2

2. **A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.**

Sol. : Given, A vessel is in the form of hollow hemisphere mounted by a hollow cylinder.

$AB = DC = 14$ cm

$\therefore OB = OC = O'P$

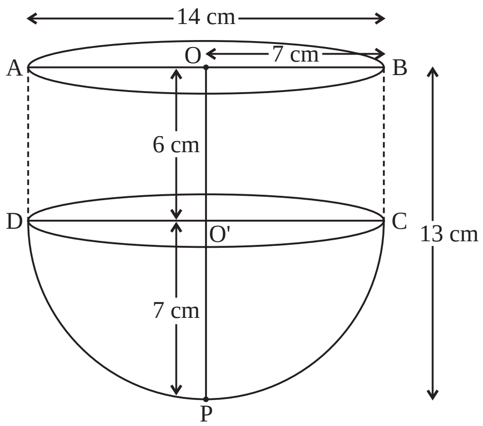
$= \frac{AB}{2} = \frac{14}{2} = 7$ cm

and $PO = 13$ cm

$\therefore OO' = PO - O'P$

$= 13 - 7 = 6$ cm

$\therefore$ Radius of the hemisphere = Height of the hemisphere = 7 cm



∴ Inner surface area of the vessel

= Curved surface area of cylinder + Curved surface area of the hemisphere

$= 2\pi rh + 2\pi r^2$

$= 2 \times \frac{22}{7} \times 7 \times 6 + 2 \times \frac{22}{7} \times 7^2$

$= 2 \times 22 \times 6 + 2 \times 22 \times 7$

$= 44[6 + 7]$

$= 44 \times 13$

$= 572 \text{ cm}^2$

3. **A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.**

Sol. : Given, A toy made up of a cone and a hemisphere. The cone is mounted on the hemisphere.

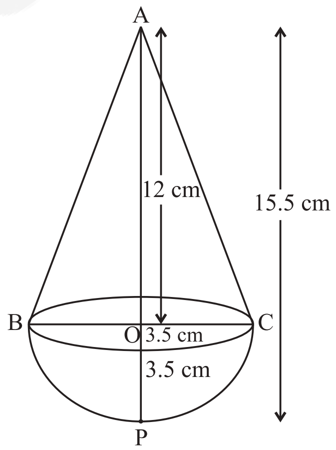
Total height of toy = 15.5 cm

and $OC = OB = OP = 3.5$ cm

∴ $AO = AP - OP$

$= 15.5 - 3.5$

$= 12$ cm



So, Radius of hemisphere = 3.5 cm

Radius of cone = 3.5 cm

and Height of Cone = 12 cm

∴ Total surface area of toy

= Curved surface area of cone + Curved surface area of hemisphere

$= \pi rl + 2\pi r^2$

$= \frac{22}{7} \times 3.5 \times 12.5 + 2 \times \frac{22}{7} \times (3.5)^2$

$= 11 \times 12.5 + 77$[$\because$ l $= \sqrt{h^2 + r^2}$

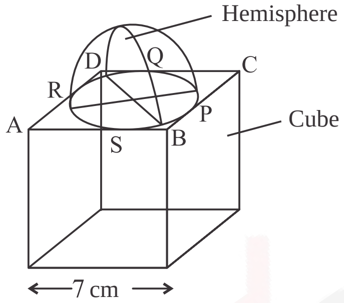
$= 137.5 + 77$ $= \sqrt{144 + 12.25}$

$= \sqrt{156.25}$ $= 214.5 \text{ cm}^2$]

$= 12.5 \text{ cm}^2$

4. **A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have ? Find the surface area of the solid.**

Sol. : Given, A cubical block is surmounted by a hemisphere hence, diameter of hemisphere should be equal to side of the cube.



$\because$ Side of cube 7 cm

$\therefore$ Radius of 5 faces hemisphere = $\frac{7}{2}$ cm

Surface area of solid = Area of 5 faces of cube+ Curved surface area of hemisphere

(Excluded the circular face *PQRS*)

+ Area of face *ABCD* of the cube

(Excluded the circular face *PQRS*)

$= 5 \times (\text{side})^2 + 2\pi r^2 + [(\text{side})^2 - \pi r^2]$

$= 5 \times 7^2 + 2 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 + \left[7^2 - \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right]$

$= 5 \times 49 + 11 \times 7 + \left[49 - \frac{77}{2}\right]$

$= 245 + 77 + 49 - \frac{77}{2}$

$= 371 - \frac{77}{2}$

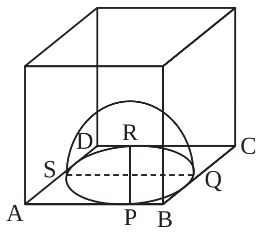
$= \frac{742 - 77}{2} = \frac{665}{2} = 332.5 \text{ cm}^2$

Hence, surface area of the solid = 332.5 cm^2

5. **A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter *l* of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.**

Sol. : Let, Side of cube= Diameter of hemisphere

= l unit



$\therefore$ Radius of hemisphere = $\frac{l}{2}$ unit

Required surface area of remaining solid

= Area of 5 faces of cube+ Area of face *ABCD* of cube– Area of hemisphere + Curved surface area of hemisphere

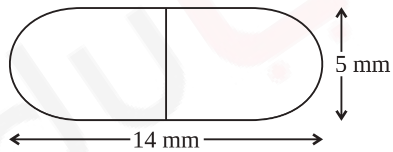
$= 5 \times (\text{side})^2 + (\text{side})^2 - \pi r^2 + 2\pi r^2$

$= 6l^2 - \frac{\pi l^2}{4} + \frac{2\pi l^2}{4} = 6l^2 + \frac{\pi l^2}{4}$

$= l^2\left[6 + \frac{\pi}{4}\right] = l^2\left(\frac{24+\pi}{4}\right)$

$= \frac{l^2}{4}(24 + \pi)$ square unit

6. **A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.**



Sol. : Given, Medicine capsule is in the shape of cylinder with two hemisphere stuck to each of its ends.

Diameter of capsule = 5 mm

$\therefore$ Radius = $\frac{5}{2}$ = 2.5 mm

and Length of capsule = 14 mm

$\therefore$ Length of cylindrical part

= 14 – (2.5 + 2.5)

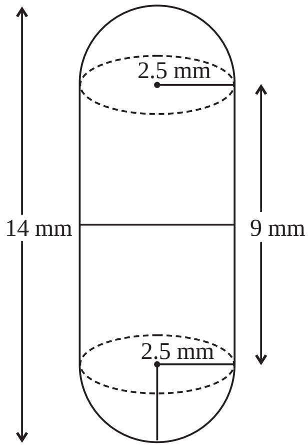
= 14 – 5 = 9 mm

Curved surface area of the hemisphere

$= 2\pi r^2$

$= 2 \times \frac{22}{7} \times 2.5 \times 2.5$

$= \frac{275}{7}$ mm^2



Curved surface area of cylinder

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times 2.5 \times 9$

$= \frac{22}{7} \times 45$

$= \frac{990}{7}$ mm^2

$\therefore$ Curved surface area of capsule

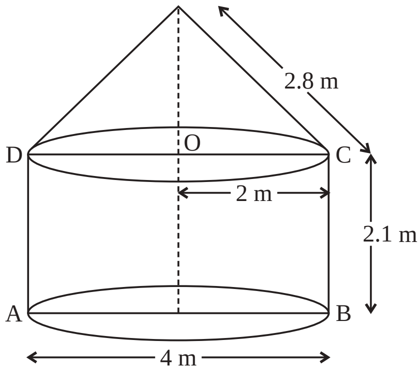
= Curved surface area of cylinder + 2 × Curved surface area of hemisphere

$= \frac{990}{7} + 2 \times \frac{275}{7} = \frac{990}{7} + \frac{550}{7}$

$= \frac{1540}{7} = 220$ mm^2

7. **A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of ₹ 500 per m^2. (Note that the base of the tent will not be covered with canvas.)**

Sol. : Given, A tent is in the shape of a cylinder surmounted by a conical top.



Given,

Slant height of cone (l) = 2.8 m

Radius of cone = $\frac{4}{2}$ = 2 m

∴ Curved surface area of cone

$= \pi rl$

$= \frac{22}{7} \times 2 \times 2.8$

$= 17.6 \text{ m}^2$

Now, height of cylinder = 2.1 m

Radius = 2 m

∴ Curved surface area of cylinder

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times 2 \times 2.1$

$= 26.4 \text{ m}^2$

∴ Area of canvas used for making the tent

= Curved surface area of cone + Curved surface area of cylinder

$= 17.6 + 26.4 = 44 \text{ m}^2$

Cost of the canvas used for making the tent at the rate of ₹ 500 per m^2

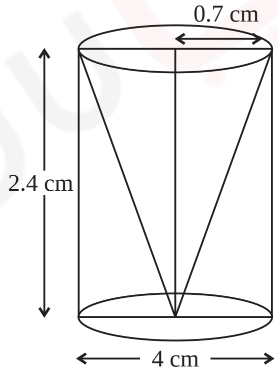
= 500 × 44 = ₹ 22000

8. **From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.**

Sol. : Given, Diameter of Cylinder

= Diameter of conical cavity

= 1.4 cm



∴ Radius of Cylinder = Radius of Conical Cavity

$= \frac{1.4}{2} = 0.7 \text{ cm}$

Height of Cylinder = Height of Conical Cavity

= 2.4 cm

Slant height of conical cavity

$= \sqrt{h^2 + r^2}$

$= \sqrt{(2.4)^2 + (0.7)^2}$

$= \sqrt{5.76 + 0.49}$

$= \sqrt{6.25} = 2.5 \text{ cm}$

Hence, total surface area of remaining solid

= Curved surface area of Conical Cavity + Curved surface area of cylinder + Base area of cylinder

$= \pi rl + (2\pi rh + \pi r^2)$

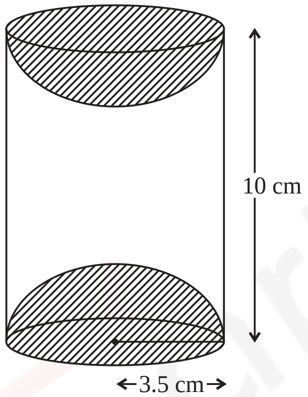
$= \pi r[l + 2h + r]$

$= \frac{22}{7} \times 0.7\ [2.5 + 2 \times 2.4 + 0.7]$

$= \frac{22}{7} \times 0.7\ (2.5 + 4.8 + 0.7)$

$= 2.2 \times 8 = 17.6 \approx 18\ cm^2$

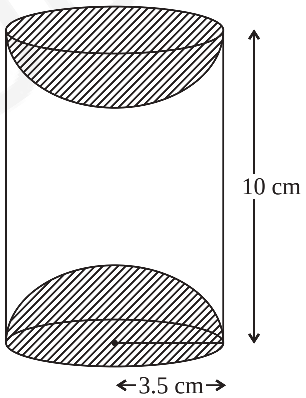
9. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in figure below. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.



Sol. : Given, Height of Cylinder = 10 cm

andbase radius of cylinder = 3.5 cm

∴ Radius of hemisphere = 3.5 cm



∴ Total surface area of the article

= 2 × Curved surface area of hemisphere + Curved surface area of cylinder

$= 2 \times (2\pi r^2) + 2\pi rh$

$= 2\pi r\ [2r + h]$

$= 2 \times \frac{22}{7} \times 3.5\ [2 \times 3.5 + 10]$

$= 22[7 + 10] = 22 \times 17 = 374\ cm^2$

Exercise – 12.2

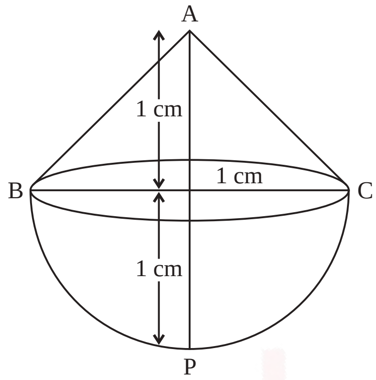
1. **A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.**

Sol. : According to the question,

Radius of cone = 1 cm

Height of cone (l) = 1 cm

Radius of hemisphere (r) = 1 cm



$\therefore$ Volume of Solid = Volume of Cone + Volume of hemisphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi \times 1^2 \times 1 + \frac{2}{3}\pi \times 1^3$$

$$= \frac{\pi}{3} + \frac{2\pi}{3} = \frac{3\pi}{3} = \pi \text{ cm}^3$$

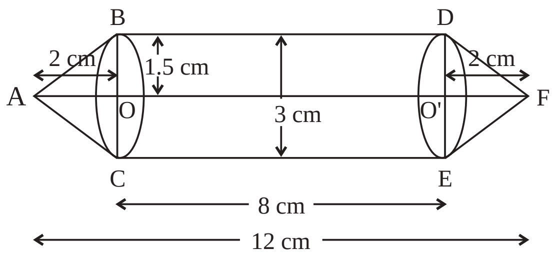
2. **Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)**

Sol. : Given, Model is made up by a cylinder and two cones

Diameter of model = 3 cm

$\therefore$ Radius $= \frac{3}{2} = 1.5$ cm

$\therefore$ Radius of Cylinder = Radius of cone = 1.5 cm



Total length of model = 12 cm

Height of Cone, h_1 = 2 cm

$\therefore$ $OO' = AE - (AO + O'F)$

$= 12 - (2 + 2)$

$OO' = 8$ cm

$\therefore$ Height of Cylinder, $h_2 = 8$ cm

$\therefore$ Volume of air contained in the model

$= 2 \times$ Volume of Cone + Volume of Cylinder

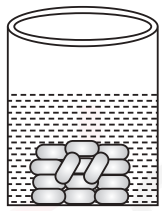
$= 2 \times \pi r^2 h_1 + \pi r^2 h_2$

$= \frac{2}{3} \times \frac{22}{7} \times 1.5 \times 1.5 \times 2 + \frac{22}{7} \times 1.5 \times 1.5 \times 8$

$= \frac{44}{21} \times 2.25 \times 2 + \frac{22}{7} \times 2.25 \times 8$

$= \frac{198}{21} + \frac{396}{21} = \frac{1386}{21} = 66 \text{ cm}^2$

3. **A gulab jamun contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm.**



Sol. : Let radius of both hemisphere and cylinder are r cm and height of cylinder is h cm which is equal to radius.

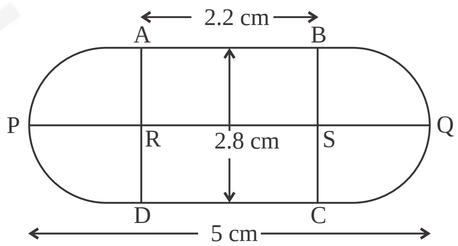
Given, diameter of each gulab jamun = 2.8 cm

$\therefore \quad r = \frac{2.8}{2} = 1.4$ cm

$\therefore \quad r = h_1 = 1.4$ cm

$h = 5 - [1.4 + 1.4]$

$= 5 - 2.8 = 2.2$ cm



$\therefore$ Volume of a gulab jamun

$= 2 \times$ Volume of hemisphere + Volume of Cylinder

$= 2 \times \frac{2}{3}\pi r^3 + \pi r^2 h$

$= \frac{4}{3} \times \frac{22}{7} \times (1.4)^3 + \frac{22}{7} \times (1.4)^2 \times 2.2$

$= \frac{22}{7} \times (1.4)^2 \left[\frac{4}{3} \times 1.4 + 2.2\right]$

$= \frac{43.12}{7}\left[\frac{5.6}{3} + 2.2\right]$

$$= \frac{43.12}{7}\left[\frac{5.6+6.6}{3}\right]$$

$$= \frac{43.12}{7}\times\frac{12.2}{3}$$

$$= \frac{526.064}{21}\ \text{cm}^3$$

∴ Volume of 45 gulab jamun

$$= 45 \times \frac{526.064}{21}$$

$$= \frac{23672.88}{21} = 1127.28$$

∵ A gulab jamun contains sugar syrup upto about 30% of its volume.

Then, quantity of sugar syrup in 1 gulab jamun.

$$= \frac{526.064}{21} \times 30\%$$

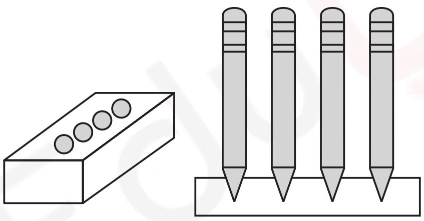
∴ Quantity of sugar syrup in 45 gulab jamun

$$= 45 \times \frac{526.064}{21}\times\frac{30}{100}$$

$$= 1127.28 \times \frac{30}{100}$$

$$= 338.18 \approx 338$$

4. **A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand.**



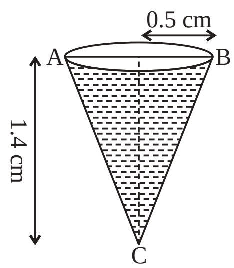
Sol. : Given, Length of Cuboid = 15 cm, Breadth of Cuboid = 10 cm and Height of cuboid = 3.5 cm

∴ Volume of cuboid = $l \times b \times h$

$$= 15 \times 10 \times 3.5 = 525\ \text{cm}^3$$

Radius of conical depression (r) = 0.5 cm

Depth of depression (h) = 1.4 cm



∴ Volume of conical depression

$$= \frac{1}{3}\pi \times r^2 \times h$$

$= \frac{1}{3} \times \frac{22}{7} \times (0.5)^2 \times 1.4$

$= \frac{1}{3} \times \frac{22}{7} \times 0.25 \times 1.4$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{25}{100} \times \frac{14}{10}$

$= \frac{11}{30}$ cm^3

$\therefore$ Volume of four conical depression

= 4 × Volume of one conical depression

$= 4 \times \frac{11}{30} = \frac{22}{15}$ cm^3

Volume of wood in the entire stand

= Volume of Cuboid – Volume of four conical depression

$= 525 - \frac{22}{15} = 525 - 1.47$

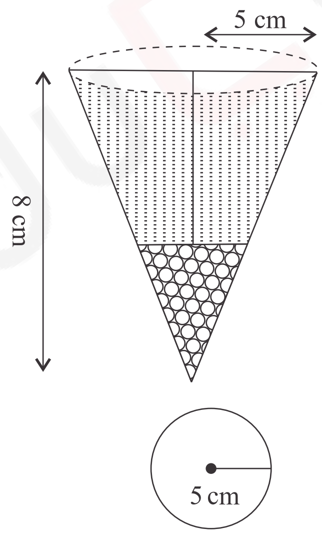
= 523.53 cm^3

5. **Vessel is in the form of an inverted cone. Its height is 8 cm and radius of its top, which is open, is 5 cm. It is filled with water upto the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one fourth of the water flows out. Find the number of lead shots dropped in the vessel.**

Sol. : Given, Vessel is in the form of an inverted cone.

Height of vessel (Cone) (h) = 8 cm

Radius of vessel (r) = 5 cm



$\therefore$ Volume of vessel filled with water

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 8$

$= \frac{4400}{21}$ cm^3

Given, Radius of a lead shots = 0.5 cm

$\therefore$ Volume of a lead shot $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times (0.5)^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 0.5$$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{11}{21} \text{cm}^3$$

Let n lead shots are dropped in the vessel.

According to the question, n × volume of a lead shot

$$= \frac{1}{4} \times \text{Volume of vessel filled with water}$$

[∵ When the lead shots are dropped in a vessel with water, then water equal to the volume of the lead shot flows out.]

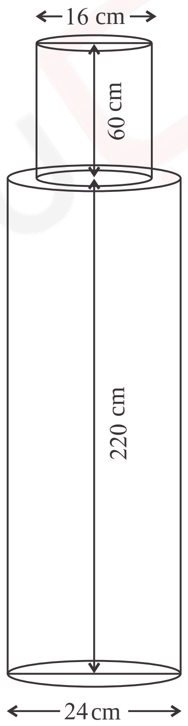
$$n \times \frac{11}{21} = \frac{4400}{21} \times \frac{1}{4}$$

$$n = \frac{1100}{11} = 100$$

∴ Number of lead shots dropped in the vessel = 100

6. **A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm³ of iron has approximately 8g mass. (Use π = 3.14)**

Sol. : Given, height of first cylinder h_1 = 220 cm



and Radius $r_1 = \frac{24}{2} = 12$ cm

∴ Volume of First Cylinder

$$= \pi r_1^2 h_1$$

$$= 3.14 \times 12 \times 12 \times 220$$

$= 3.14 \times 144 \times 220$

$= 3.14 \times 31680 \text{ cm}^3$

Height of Second Cylinder

$h_2 = 60$ cm

Radius $r_2 = 8$ cm

Volume of Second Cylinder

$= \pi r_2^2 h_2$

$= 3.14 \times 8 \times 8 \times 60$

$= 3.14 \times 64 \times 60$

$= 3.14 \times 3840 \text{ cm}^3$

$\therefore$ Volume of iron pole = Volume of First Cylinder + Volume of Second Cylinder

$= 3.14 \times 31680 + 3.14 \times 3840$

$= 3.14[31680 + 3840]$

$= 3.14 \times 35520 \text{ cm}^3$

Given,

Mass of 1 cm^3 of iron = 8 gm = $\frac{8}{1000}$ kg

$\therefore$ Mass of $3.14 \times 35520 \text{ cm}^3$ of iron

$= \frac{8}{1000} \times 3.14 \times 35520$

$= \frac{8}{1000} \times 111532.8$

$= \frac{892262.4}{1000}$

$= 892.2624$

$= 892.26$ kg

7. **A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.**

Sol. : Given, Height of Cylinder (h) = 180 cm = 1.8 m

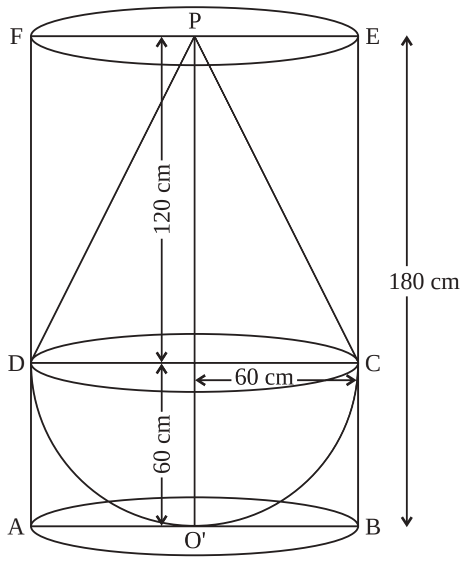
Radius of Cylinder = 60 cm = 0.6 m [$\because$ 1 cm = $\frac{1}{100}$ m]

Volume of water filled in circular cylinder = $\pi r^2 h$

$= \frac{22}{7} \times 0.6 \times 0.6 \times 1.8$

$= \frac{14.256}{7} \text{ m}^3$

A solid consisting of right circular cone standing on the hemisphere.



Height of cone (h_1) = 120 cm = 1.2 m

Radius of cone (r_1) = 60 cm = 0.6 m

Radius of hemisphere r_2 = 60 cm = 0.6 m

$\therefore$ Volume of Solid = Volume of Cone + Volume of hemisphere

$$= \frac{1}{3}\pi r_1^2 h + \frac{2}{3}\pi r_2^3$$

$$= \frac{1}{3}\times\frac{22}{7}\times(0.6)^2\times1.2+\frac{2}{3}\times\frac{22}{7}\times(0.6)^3$$

$$= \frac{22}{21} \times 0.36 \times 1.2 + \frac{22}{21} \times 2 \times 0.36 \times 0.6$$

$$= \frac{22}{21} \times 0.36[1.2 + 1.2]$$

$$= \frac{22}{21} \times 0.36 \times 2.4$$

$$= \frac{19.008}{21} = \frac{6.336}{7} \text{m}^3$$

Hence, volume of water left in the cylinder

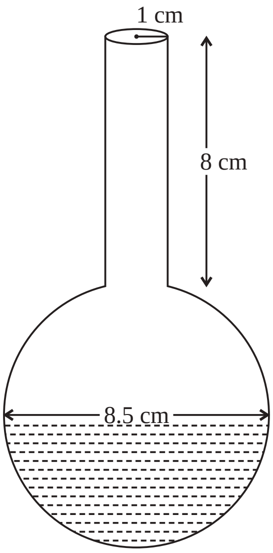
= Volume of water filled in cylinder – Volume of solid

$$= \frac{14.256}{7} - \frac{6.336}{7} = \frac{7.92}{7} = 1.131 \text{ m}^3$$

8. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and π = 3.14.

Sol. : It is not correct.

Given : A spherical glass vessel has a cylindrical neck



Radius of Cylinder, $r_1 = \frac{2}{2} = 1$ cm

Height of Cylinder, $h = 8$ cm

Radius of Sphere, $r_2 = \frac{8.5}{2}$ cm

$\therefore$ Volume of water filled in the spherical glass

$= \text{Volume of Cylinder} + \text{Volume of Sphere}$

$= \pi r_1^2 h_1 + \frac{4}{3}\pi r_2^3$

$= 3.14 \times 1\times 1 \times 8 + \frac{4}{3} \times 3.14 \times \frac{8.5}{2}\times\frac{8.5}{2}\times\frac{8.5}{2}$

$= 25.12 + \frac{1928.3535}{6}$

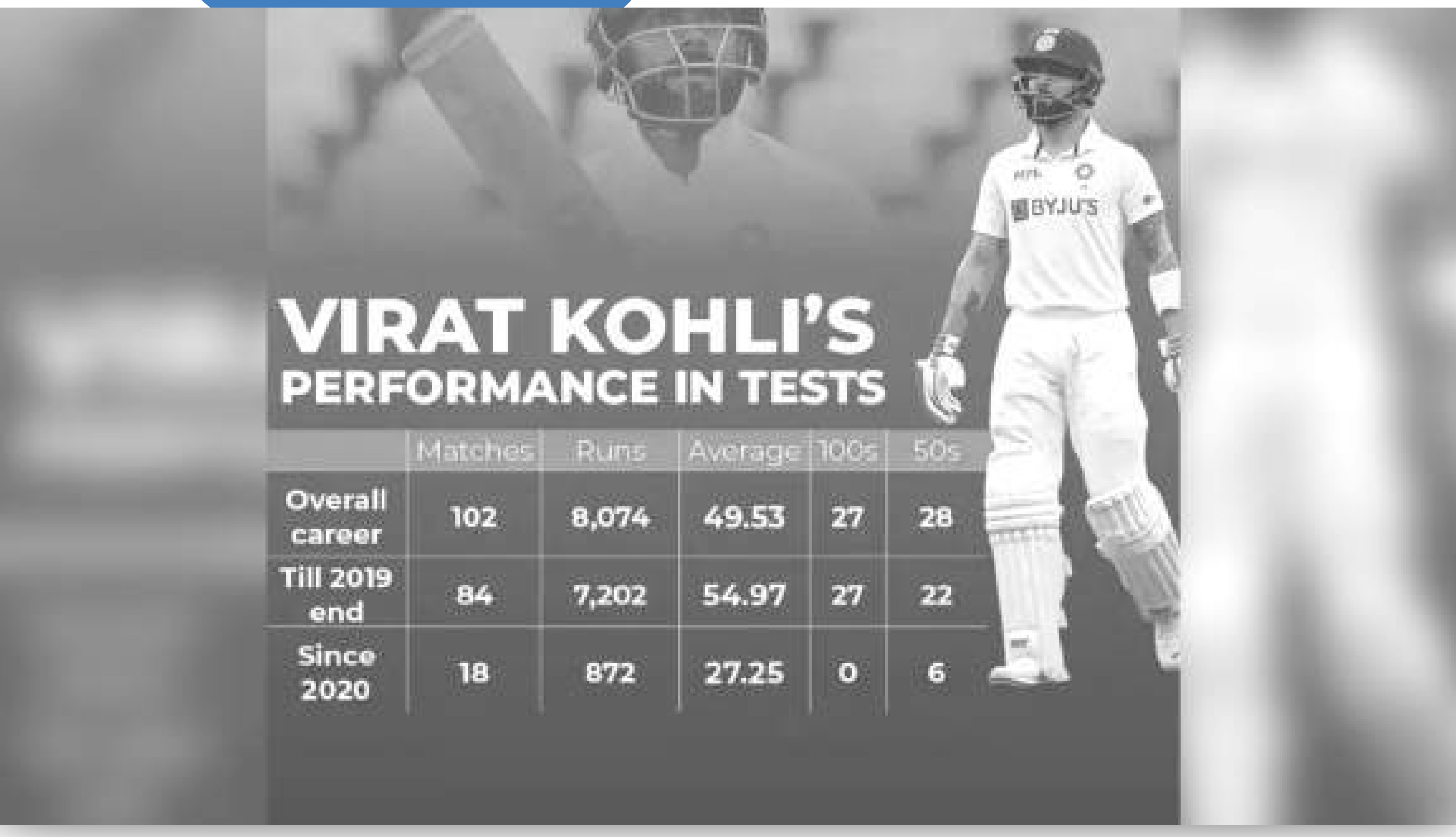
$= 25.12 + 321.39$

$= 346.51 \text{ cm}^3$

Hence, correct answer is 346.51 cm^3.

Statistics 13

NCERT SOLUTIONS



What's inside

– *In-Chapter Q's (solved)*

– *Textbook Exercise Q's (solved)*

Exercise – 13.1

1. A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants	**0-2**	**2-4**	**4-6**	**6-8**	**8-10**	**10-12**	**12-14**
Number of houses	**1**	**2**	**1**	**5**	**6**	**2**	**3**

Which method did you use for finding the mean, and why?

Sol. : We will use the direct method to find the mean of the data because the number are small.

Class Interval	**Mean Value (x_i)**	**Frequency (f_i)**	$f_i x_i$
0-2	1	1	1
2-4	3	2	6
4-6	5	1	5
6-8	7	5	35
8-10	9	6	54
10-12	11	2	22
12-14	13	3	39
Total		$\Sigma f_i = 20$	$\Sigma f_i x_i = 162$

Here, $\Sigma f_i = 20$ and $\Sigma f_i x_i = 162$

$$\therefore \quad \text{Arithmetic Mean} = \frac{\sum_{n=1}^{n} f_i x_i}{\sum_{n=1}^{n} f_i} = \frac{162}{20} = 8.1$$

Hence, required value of data is 8.1

Here, we will use the direct method to find the mean of the data because the value of x_i and f_i are small.

2. Consider the following distribution of daily wages of 50 workers of a factory :

Daily wages (in ₹)	**500-520**	**520-540**	**540-560**	**560-580**	**580-600**
Number of workers	**12**	**14**	**8**	**6**	**10**

Find the mean daily wages of the workers of the factory by using an appropriate method.

Sol. : Here, we will use step deviation method to find the mean of data.

Daily Wages (In ₹)	Mean Value (x_i)	No. of Workers (f_i)	$d_i = x_i - a$	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
500-520	510	12	– 40	– 2	– 24
520-540	530	14	– 20	– 1	– 14
540-560	550 = a	8	0	0	0
560-580	570	6	20	1	6
580-600	590	10	40	2	20
Total		Σf_i = **50**			$\Sigma f_i u_i$ = **– 12**

Let assumed mean (a) = 550,

Class width h = (520 – 500) = 20

$\Sigma f_i = 50$, $\Sigma f_i u_i = -12$

By Step deviation method,

$$\bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$= 550 + 20\left(\frac{-12}{50}\right)$$

$$= 550 - \frac{24}{5}$$

$$= 550 - 4.8$$

$$= 545.20$$

3. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is ₹ 18. Find the missing frequency *f*.

Daily Pocket Allowance (In ₹)	No. of Children
11-13	**7**
13-15	**6**
15-17	**9**
17-19	**13**
19-21	***f***
21-23	**5**
23-25	**4**

Sol. : The arithmetic mean is following :

Daily Pocket Allowance (In ₹)	Mean Value (x_i)	No. of Children (f_i)	$d_i = x_i - a$	$f_i d_i$
11-13	12	7	– 6	– 42
13-15	14	6	– 4	– 24
15-17	16	9	– 2	– 18

Daily Pocket Allowance (In ₹)	Mean Value (x_i)	No. of Children (f_i)	$d_i = x_i - a$	f_id_i
17-19 19-21 21-23 23-25	18 = a 20 22 24	13 f 5 4	0 2 4 6	0 $2f$ 20 24
Total		$\Sigma f_i = 44 + f$		$\Sigma f_id_i = 2f - 40$

Let assumed mean (a) = 18

Given, Mean = ₹ 18

From table $\Sigma f_i = 44 + f$ and $f_id_i = 2f - 40$

By Step deviation method,

$$\overline{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} = 18 + \frac{2f - 40}{44 + f}$$

$\therefore$ $$18 = 18 + \frac{2f - 40}{44 + f}$$

$\Rightarrow$ $$\frac{2f - 40}{44 + f} = 0 \Rightarrow 2f - 40 = 0$$

$$f = \frac{40}{2} = 20$$

4. 30 women were examined in a hospital by a doctor and the number of heartbeats per minute were recorded and summarised as follows. Find the mean heartbeats per minute for these women, choosing a suitable method.

Number of heartbeats per minute	Number of women
65-68	**2**
68-71	**4**
71-74	**3**
74-77	**8**
77-80	**7**
80-83	**4**
83-86	**2**

Sol. : Here, we will use step deviation method to find the mean of data.

Assumed mean (a) = 75·5,

Class width h = 68 – 65 = 3

Following is the table for arithmetic mean :

Number of heartbeats per minute	Mean Value x_i	No. of women (f_i)	$d_i = x_i - a$	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
65-68	66.5	2	– 9	– 3	– 6
68-71	69.5	4	– 6	– 2	– 8
71-74	72.5	3	– 3	– 1	– 3
74-77	75.5 = a	8	0	0	0
77-80	78.5	7	3	1	7
80-83	81.5	4	6	2	8
83-86	84.5	2	9	3	6
		$\Sigma f_i = 30$			$\Sigma f_i u_i = 4$

Here, $\Sigma f_i = 30$ and $\Sigma f_i u_i = 4$

By Step deviation method,

$$\bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$= 75.5 + 3\left(\frac{4}{30}\right) = 75.5 + 0.4$$

Mean = 75.9 Heartbeats

5. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes :

Number of Mangoes	50-52	53-55	56-58	59-61	62-64
Number of boxes	15	110	135	115	25

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

Sol. : The given data is not continuous so we add 0.5 to the upper limit of each class and subtract 0.5 to the lower limit.

Number of Mangoes	Mean Value (x_i)	Number of boxes (f_i)	$d_i = x_i - a$	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
49.5-52.5	51	15	– 6	– 2	– 30
52.5-55.5	54	110	– 3	– 1	–110
55.5-58.5	57 = a	135	0	0	0
58.5-61.5	60	115	3	1	115
61.5-64.5	63	25	6	2	50
		400			**25**

Let assumed mean (a) = 57,

Class width $h = 52{\cdot}5 - 49{\cdot}5 = 3$

Here, $\Sigma f_i = 400$, $\Sigma f_i u_i = 25$

By Step deviation method,

$$\bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$= 57 + 3\left(\frac{25}{400}\right) = 57 + \frac{75}{400}$$

$= 57 + 0.19 = 57.19$

6. The table below shows the daily expenditure on food of 25 households in a locality :

Daily expenditure (in ₹)	100-150	150-200	200-250	250-300	300-350
Number of household	4	5	12	2	2

Find the mean daily expenditure on food by a suitable method.

Sol. : Following is the table for calculating arithmetic mean

Daily expenditure (In ₹)	Mean Value (f_i)	Number of household (f_i)	$d_i = x_i - a$	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
100-150	125	4	– 100	– 2	– 8
150-200	175	5	– 50	– 1	– 5
200-250	225 = a	12	0	0	0
250-300	275	2	50	1	2
300-500	325	2	100	2	4
		$\Sigma f_i = 25$			$\Sigma f_i u_i = -7$

Let assumed mean (a) = 225,

Class width (h) = 150 – 100 = 50

$\Sigma f_i = 25$, $\Sigma f_i u_i = -7$

$\therefore$ By Step deviation method,

$$\overline{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) = 225 + 50\left(\frac{-7}{25}\right) = 225 - 7 \times 2$$

$= 225 - 14 = 211$

Hence, mean expenditure on food is ₹ 211.

7. To find out the concentration of SO_2 in the air (in parts per million, *i.e.*, ppm), the data was collected for 30 localities in a certain city and is presented below :

Concentration of SO_2 (in ppm)	Frequency
0.00-0.04	**4**
0.04-0.08	**9**
0.08-0.12	**9**
0.12-0.16	**2**
0.16-0.20	**4**
0.20-0.24	**2**

Find the mean concentration of SO_2 in the air.

Sol. :

Concentration of SO_2 (in ppm)	Frequency (f_i)	Mean Value (x_i)	$d_i = x_i - a$	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
0.00-0.04	4	0·02	– 0.08	– 2	– 8
0.04-0.08	9	0·06	– 0.04	– 1	– 9
0.08-0.12	9	0·10 = a	0	0	0

0.12-0.16	2	0.14	0.04	1	2
0.16-0.20	4	0.18	0.08	2	8
0.20-0.24	2	0.22	0.12	3	6
Total	$\Sigma f_i = 30$				$\Sigma f_i u_i = -1$

Here, assumed mean (a) = 0.10,

Class width (h) = 0.04 – 0.00 = 0.04

By Step deviation,

$$\text{Mean } \bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$= 0.10 + 0.04\left(\frac{-1}{30}\right)$$

$$= 0.10 - 0.001$$

$$= 0.099 \text{ ppm}$$

8. A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

Number of days	**0-6**	**6-10**	**10-14**	**14-20**	**20-28**	**28-38**	**38-40**
Number of students	**11**	**10**	**7**	**4**	**4**	**3**	**1**

Sol. :

Number of days	**Mean Value x_i**	**Number of students (f_i)**	$d_i = x_i - a$	$f_i \times d_i$
0-6	3	11	– 14	– 154
6-10	8	10	– 9	– 90
10-14	12	7	– 5	– 35
14-20	17 = a	4	0	0
20-28	24	4	7	28
28-38	33	3	16	48
38-40	39	1	22	22
Total		$\Sigma f_i = 40$		$\Sigma f_i d_i = -181$

Here, assumed mean (a) = 17,

$\Sigma f_i = 40$ and $\Sigma f_i d_i = -181$

By Step deviation,

$$\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} = 17 + \left(\frac{-181}{40}\right)$$

$$= 17 - 4.52 = 12.48 \text{ days}$$

9. **The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.**

Literacy Rate (in %)	45-55	55-65	65-75	75-85	85-95
Number of cities	3	10	11	8	3

Sol. :

Literacy Rate (In %)	Number of cities (f_i)	Mean Value (x_i)	$d_i = x_i - a$	$u_i = \frac{x_i - a}{h}$	$f_i \times u_i$
45-55	3	50	– 20	– 2	– 6
55-65	10	60	– 10	– 1	– 10
65-75	11	70 = a	0	0	0
75-85	8	80	10	1	8
85-95	3	90	20	2	6
Total	$\Sigma f_i = 35$				$\Sigma f_i u_i = -2$

Let assumed mean (a) = 70,

Class width $h = 55 - 45 = 10$

$\Sigma f_i = 35$ and $\Sigma f_i u_i = -2$

By Step deviation method,

$$\text{Mean } \bar{x} = a + h\left(\frac{\Sigma f_i \times u_i}{\Sigma f_i}\right) = 70 + 10\left(\frac{-2}{35}\right)$$

$$= 70 - 0.57 = 69.43\%$$

Exercise – 13.2

1. **The following table shows the ages of the patients admitted in a hospital during a year :**

Age (in years)	Number of patients
5-15	**6**
15-25	**11**
25-35	**21**
35-45	**23**
45-55	**14**
55-65	**5**

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Sol. :

Age (in years)	Mean Value (x_i)	Number of patients	$d_i = x_i - a$	$u_i = \frac{x_i - a}{h}$	$f_i \times x_i$
5-15	10	6	– 20	– 2	– 12
15-25	20	11	– 10	– 1	– 11

Age (in years)	Mean Value (x_i)	Number of patients	$d_i = x_i - a$	$u_i = \frac{x_i - a}{h}$	$f_i \times x_i$
25–35	$30 = a$	21	0	0	0
35-45	40	23	10	1	23
45-55	50	14	20	2	28
55-65	60	5	30	3	15
Total		$\Sigma f_i = 80$			$\Sigma f_i u_i = 43$

Here, assumed mean (a) = 30,

Class width $h = 15 - 5 = 10$

$\Sigma f_i = 80$, $\Sigma f_i u_i = 43$

Modal class from the given data = 35–45 [$\because$ Class interval has maximum frequency]

Lower limit of modal class (l) = 35

$f_1 = 23$, $f_0 = 21$, $f_2 = 14$ and Class width $h = 10$

Hence, Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$$= 35 + \left(\frac{23 - 12}{2 \times 23 - 21 - 14}\right) \times 10$$

$$= 35 + \frac{20}{11} = 35 + 1.82$$

$= 36{\cdot}82$ year

Now to find the mean of the data, using Step deviation method,

$$\bar{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h$$

$$= 30 + \left(\frac{43}{80}\right) \times 10$$

$$= 30 + \frac{43}{8} = 30 + 5.38$$

= 35.38 years

Hence, Mode = 36.82 year and Mean = 35.38 years

We conclude that the maximum age of patients admitted to the hospital is approximately 36.82 years while the mean age of a pateint admitted to the hospital is 35.38 years.

2. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components :

Lifetimes (in hours)	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	**10**	**35**	**52**	**61**	**38**	**29**

Determine the modal lifetimes of the components.

Sol. : Modal class of given data is 60–80 because class interval has maximum frequency.

Here, lower limit of modal class (l) = 60,

Class width $h = 80 - 60 = 20$

$f_1 = 60$, $f_0 = 52$ and $f_2 = 38$

$\because$ Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 60 + \left(\frac{61 - 52}{2 \times 61 - 52 - 38}\right) \times 20$

$= 60 + \frac{9 \times 20}{32} = 60 + \frac{45}{8}$

$= 60 + 5.625 = 65.625$ hr.

3. The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure.

Expenditure (in ₹)	Number of families
1000-1500	**24**
1500-2000	**40**
2000-2500	**33**
2500-3000	**28**
3000-3500	**30**
3500-4000	**22**
4000-4500	**16**
4500-5000	**7**

Sol. :

Expenditure	Number of families (f_i)	Mean Value (x_i)	$d_i = x_i - a$	$u_i = \frac{x_i - a}{h}$	$f_i \times u_i$
1000-1500	24	1250	– 1500	– 3	– 72
1500-2000	40	1750	– 1000	– 2	– 80
2000-2500	33	2250	– 500	– 1	– 33
2500-3000	28	2750 = a	0	0	0
3000-3500	30	3250	500	1	30
3500-4000	22	3750	1000	2	44
4000-4500	16	4250	1500	3	48
4500-5000	7	4750	2000	4	28
Total	$\Sigma f_i = 200$				$\Sigma f_i \times u_i = -35$

Modal Class of data is 1500-2000 because class interval has maximum frequency.
Lower limit of modal class $l = 1500$
$f_1 = 40$, $f_0 = 24$, $f_2 = 33$, $h = 500$

$\therefore$ Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 1500 + \left(\frac{40 - 24}{2 \times 40 - 24 - 33}\right) \times 500$

$= 1500 + \frac{16 \times 500}{23}$

$= 1500 + \frac{8000}{23}$

$= 1500 + 347.83 = 1847.83$

Hence, assumed mean $(a) = 2750$,

Class width $(h) = 500$

Total observation $(N) = 200$ and $\Sigma f_i u_i = -35$

From Step deviation method,

$$\bar{x} = a + \left(\frac{\Sigma f_i u_i}{N}\right) \times h = 2750 + \left(\frac{-35}{200}\right) \times 500$$

$$= 2750 - \frac{35 \times 5}{2} = 2750 - 87.50$$

$= ₹\ 2662.50$

Hence, the maximum mode monthly expenditure is 1847.83 and the mean monthly expenditure is 2662.50.

4. The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures.

No. of students per teacher	No. of state/U.T.
15-20	**3**
20-25	**8**
25-30	**9**
30-35	**10**
35-40	**3**
40-45	**0**
45-50	**0**
50-55	**2**

Sol. :

Number of students per teacher	No. of State/ U.T. (f_i)	Mean Value (x_i)	$d_i = x_i - a$	$u_i = \frac{x_i - a}{h}$	$f_i \times x_i$
15-20	3	17.5	– 15	– 3	– 9
20-25	8	22.5	– 10	– 2	– 16
25-30	9	27.5	– 5	– 1	– 9
30-35	10	32.5 = a	0	0	0
35-40	3	37.5	5	1	3
40-45	0	42.5	10	2	0
45-50	0	47.5	15	3	0
50-55	2	52.5	20	4	8
Total	$\Sigma f_i =$ 35				$\Sigma f_i u_i =$ – 23

Sol. : Modal class of data is 30-35 because the class interval has maximum frequency.

$\therefore$ Lower limit of modal class $l = 30$, $f_1 = 10$, $f_0 = 9$, $f_2 = 3$ and Class width $(h) = 5$

$$\therefore \quad \text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 30 + \left(\frac{10 - 9}{2 \times 10 - 9 - 3}\right) \times 5$$

$$= 30 + \frac{5}{8} = 30 + 0.63 = 30.63$$

Now, assumed mean $(a) = 32 \cdot 5$, Total observation $N = 35$

and $\sum f_i \times u_i = -23$, Class width $h = 5$

By Step deviation method,

$$\bar{x} = a + \frac{\Sigma f_i u_i}{N} \times h$$

$$= 32.5 + \frac{-23}{35} \times 5 = 32.5 - \frac{23}{7}$$

$$= 32.5 - 3.3 = 29.2$$

Hence, Mode = 30.6 and Mean ($\bar{x}$) = 29.2

Hence, it is concluded that the teacher-student ratio in most of the states and union territories is 30.6 and the mean is 29.2.

5. The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches.

Runs scored	Number of batsmen
3000-4000	4
4000-5000	18
5000-6000	9
6000-7000	7
7000-8000	6
8000-9000	3
9000-10000	1
10000-11000	1

Find the mode of the data.

Sol. : $\because$ Maximum frequency $(f_1) = 18$. So, modal class interval is 4000-5000.

$\therefore$ Lower limit of Modal class $l = 4000$,

Class width $h = 1000$, $f_1 = 18$, $f_0 = 4$, $f_2 = 9$

$$\text{Hence,} \quad \text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 4000 + \left(\frac{18 - 4}{2 \times 18 - 4 - 9}\right) \times 1000$$

$$= 4000 + \frac{14000}{23}$$

= 4000 + 608.7 = 4608.7 Runs

6. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data :

No. of cars	Frequency
0-10	7
10-20	14
20-30	13
30-40	12
40-50	20
50-60	11
60-70	15
70-80	8

Sol. : $\because$ Maximum of frequency (f_1) = 20, hence, modal class interval is (40–50)

Lower limit of modal class (l) = 40, Class width (h) = 10

$f_1 = 20$, $f_0 = 12$ and $f_2 = 11$

Hence, Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$$= 40 + \left(\frac{20 - 12}{2 \times 20 - 12 - 11}\right) \times 10$$

$$= 40 + \frac{80}{17} = 40 + 4.7$$

= 44.7 Cars

Exercise – 13.3

1. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly Consumption (In Units)	Number of Consumers
65-85	4
85-105	5
105-125	13
125-145	20
145-165	14
165-185	8
185-205	4

Sol. :

Monthly Consumption (x)	Number of Consumers (f)	Cumulative Frequency (cf)
65-85	4	4
85-105	5	9

105-125	13	22 = cf
125-145	20 = f	42
145-165	14	56
165-185	8	64
185-205	4	68

(i) Here, $N = 68$ (which is an even number)

$$\frac{N}{2} = \frac{68}{2} = 34$$

So, the median class is (125–145)

$\therefore$ Lower limit of modal class (l) = 125, N = 68, f = 20, cf = 22, Class width (h) = 20

$$\therefore \quad \text{Median} = l + \left\{\frac{\left(\frac{N}{2} - cf\right)}{f}\right\} \times h$$

$$= 125 + \left(\frac{34 - 22}{20}\right) \times 20$$

= 125 + 12 = 137 Units

(ii) $\because$ Maximum frequency f_m = 20. So, its modal class is (125–145)

f_1 = 13, f_2 = 14, lower limit of modal class (l) = 125 and Class width (h) = 20

$$\therefore \quad \text{Mode} = l + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right) \times h$$

$$= 125 + \left(\frac{20 - 13}{2 \times 20 - 13 - 14}\right) \times 20$$

$$= 125 + \frac{7 \times 20}{13} = 125 + \frac{140}{13}$$

= 125 + 10.76 = 135.76 Unit

(iii)

Monthly Consump-tion	Number of Consumers (f_i)	Mean Value (x_i)	$u_i\, x_i - a$	$u_i = \frac{x_i - a}{h}$	$f_i\, u_i$
65-85	4	75	– 60	– 3	– 12
85-105	5	95	– 40	– 2	– 10
105-125	13	115	– 20	– 1	– 13
125-145	20	135 = a	0	0	0
145-165	14	155	20	1	14
165-185	8	175	40	2	16
185-205	4	195	60	3	12
Total	$\Sigma f_i = 68$				$\Sigma f_i u_i = 7$

Total observation (N) = 68, Assumed mean (a) = 135

Class width (h) = 20 and $\Sigma f_i u_i$ = 7

$\therefore$ Mean = $a+\left(\frac{\Sigma f_i.u_i}{N}\right)\times h$

$= 135+\left(\frac{7}{68}\right)\times 20 = 135+\frac{35}{17}$

= 135 + 2·05 = 137·05 Units

2. If the median of the distribution given below is 28.5, find the values of *x* and *y*.

Class-Interval	Frequency
0-10	**5**
10-20	***x***
20-30	**20**
30-40	**15**
40-50	***y***
50-60	**5**
Total	**60**

Sol. :

Class-Interval	Frequency (f)	Cumulative Frequency
0-10	5	5
10-20	x	$5 + x = cf$
20-30	$20 = f$	$25 + x$
30-40	15	$40 + x$
40-50	y	$40 + x + y$
50-60	5	$45 + x + y$
Total	60	

Given, Median = 28.5 which falls in the class interval (20–30).

Hence, lower limit of median class (l) = 20, f = 20, cf = 5 + x

Class width (h) = 10, Total observation N = 60

$\therefore$ Median = $l+\left(\frac{\frac{N}{2}-cf}{f}\right)\times h$

Or 28.5 = $20+\frac{\left(\frac{60}{2}-(5+x)\right)}{20}\times 10$

Or 8.5 = $\frac{30-(5+x)}{2}$

Or 17 = 25 – x

$\therefore$ x = 25 – 17 = 8

From the given table,

$45 + x + y = 60$

Or $45 + 8 + y = 60$

Or $y = 60 - 53$

$\therefore$ $y = 7$

So, the value of x and y are 8 and 7 respectively.

3. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 years.

Age (in years)	Number of policy holders
Below 20	**2**
Below 25	**6**
Below 30	**24**
Below 35	**45**
Below 40	**78**
Below 45	**89**
Below 50	**92**
Below 55	**98**
Below 60	**100**

Sol. :

Age (in years) (x)	Number of Policy Holders (f)	Cumulative Frequency (cf)
Below 20	2	2
20-25	6 – 2 = 4	6
25-30	24 – 6 = 18	24
30-35	45 – 24 = 21	45 = cf
35-40	78 – 45 = 33 = f	78
40-45	89 – 78 = 11	89
45-50	92 – 89 = 3	92
50-55	98 – 92 = 6	98
55-60	100 – 98 = 2	100
Total	N = 100	

$$\frac{N}{2} = \frac{100}{2} = 50$$

Here, Lower limit of modal class (l) = 35, f = 33, cf = 45, Class width (h) = 5

$$\text{Median} = l + \left\{\frac{\left(\frac{N}{2} - cf\right)}{f}\right\} \times h$$

$$= 35 + \left(\frac{50 - 45}{33}\right) \times 5$$

$$= 35 + \frac{25}{33} = 35 + 0.76 = 35.76 \text{ yr.}$$

4. **The lengths of 40 leaves of a plant are measured correct to the nearest millimetre, and the data obtained is represented in the following table :**

Length (in mm)	Number of leaves
118-126	**3**
127-135	**5**
136-144	**9**
145-153	**12**
154-162	**5**
163-171	**4**
172-180	**2**

Find the median length of the leaves.

Sol. : Since the data is not continuous so we need to convert it into continuous to find the median then the class 117.5-126.5, 126.5-135.5, ..., 175.5-180.5.

Length (In mm)	No. of leaves	Cumulative Frequency
117.5-126.5	3	3
126.5-135.5	5	8
135.5-144.5	9	17 = cf
144.5-153.5	12 = f	29
153.5-162.5	5	34
162.5-171.5	4	38
171.5-180.5	2	40

$$\frac{N}{2} = \frac{40}{2} = 20$$

∵ Cumulative frequency 20, Median Class interval = 144.5–153.5.

Here, Lower limit of median class (l) = 144.5

$f = 12$, $cf = 17$

Class width (h) = 9

Total Observation (N) = 40

$$\text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$= 144.5 + \frac{\left(\frac{40}{2} - 17\right)}{12} \times 9$$

$$= 145.5 + \left(\frac{20 - 17}{12}\right) \times 9$$

$$= 144.5 + \frac{3}{12} \times 9$$

$$= 144.5 + \frac{9}{4} = 144.5 + 2.25$$

$$= 146.75 \text{ mm}$$

Hence, median length of leaves is 146.75 mm.

5. The following table gives the distribution of the life time of 400 neon lamps :

Life times (in hours)	Number of lamps
1500-2000	14
2000-2500	56
2500-3000	60
3000-3500	86
3500-4000	74
4000-4500	62
4500-5000	48

Find the median life time of a lamp.

Sol. :

Life Time (in hours)	Number of lamps	Cumulative Frequency
1500-2000	14	14
2000-2500	56	70
2500-3000	60	130 = cf
3000-3500	86 = f	216
3500-4000	74	290
4000-4500	62	352
4500-5000	48	400
Total	N = 400	

$$\frac{N}{2} = \frac{400}{2} = 200$$

Cumulative frequency 200, falls in the class interval 3000-3500.

Here, Lower limit of median class (l) = 3000, f = 86, cf = 130

Class width h= 500

Total observation N = 400

$$\therefore \quad \text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$= 3000 + \left(\frac{200 - 130}{86}\right) \times 500$$

$$= 3000 + \frac{70 \times 500}{86}$$

$$= 3000 + \frac{35000}{86}$$

$$= 3000 + 406.98 = 3406.98 \text{ hr.}$$

Hence, the median life time of a lamp is 3406.98 hours.

6. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows :

Number of letters	1-4	4-7	7-10	10-13	13-16	16-19
Number of surnames	6	30	40	16	4	4

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames. Also, find the modal size of the surnames.

Sol. :

Number of Letters	Number of Surnames	Cumulative Frequency
1-4	6	6
4-7	30	6 + 30 = 36 = *cf*
7-10	40 = *f*	36 + 40 = 76
10-13	16	76 + 16 = 92
13-16	4	92 + 4 = 96
16-19	4	96 + 4 = 100
Total	*N* = 100	

(i) $\frac{N}{2} = \frac{100}{2} = 50$

$\because$ Cumulative frequency 50, locate in the class interval (7–10).

Here, Lower limit of median class (l) = 7, f = 40, cf = 36

Class width (h) = 3, Total observation N = 100

$$\therefore \quad \text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$= 7 + \left(\frac{50 - 36}{40}\right) \times 3 = 7 + \frac{14 \times 3}{40}$$

$$= 7 + \frac{21}{20} = 7 + 1{\cdot}05 = 8{\cdot}05$$

The median number of letters in the surnames is 8.05.

(ii) $\because$ Modal Class is (7–10) because its maximum frequency (f_m) = 40.

Lower limit of modal class (l) = 7, f_m = 40, f_1 = 30, f_2 = 16

Class width (h) = 3

$$\therefore \quad \text{Mode} = l + \left(\frac{f_m - f_l}{2f_m - f_1 - f_2}\right) \times h$$

$$= 7 + \left(\frac{40 - 30}{2 \times 40 - 30 - 16}\right) \times 3$$

$$= 7 + \frac{10 \times 3}{34} = 7 + 0.88 = 7.088$$

Hence, modal size of the surnames = 7.088.

(iii) We know that,

$$\text{Mode} = 3\,(\text{Median}) - 2\,(\text{Mean})$$

Or $\quad 7.088 = 3 \times 8.05 - 2\,(\text{Mean})$

Or $\quad 2(\text{Mean}) = 24.15 - 7.088$

$\therefore \quad \text{Mean} = \frac{17.062}{2} = 8.53$

7. The distribution below gives the weights of 30 students of a class. Find the median weight of the students.

Weight (in kg)	Number of students
40-45	**2**
45-50	**3**
50-55	**8**
55-60	**6**
60-65	**6**
65-70	**3**
70-75	**2**

Sol. :

Weight (in kg)	Number of students	Cumulative Frequency
40-45	2	2
45-50	3	2 + 3 = 5
50-55	8	5 + 8 = 13 = cf
55-60	6 = f	13 + 6 = 19
60-65	6	19 + 6 = 25
65-70	3	25 + 3 = 28
70-75	2	28 + 2 = 30
Total	$N = 30$	

$$\frac{N}{2} = \frac{30}{2} = 15$$

$\because$ Cumulative frequency 15, locate in the class interval (55–60).
Here, Lower limit of median class, $(l) = 55$, $f = 6$, $cf = 13$
Class width $(h) = 5$, Total observation $(N) = 30$

$$\therefore \quad \text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$= 55 + \left(\frac{15 - 13}{6}\right) \times 5 = 55 + \frac{2}{6} \times 5$$

$$= 55 + \frac{5}{3} = 55 + 1.67$$

$$= 56.67$$

Hence, the median weight of the student is 56.67 kg.

Probability 14

NCERT SOLUTIONS

What's inside

– *In-Chapter Q's (solved)*
– *Textbook Exercise Q's (solved)*

Exercise – 14.1

1. **Complete the following statements :**

(i) Probability of an event E + Probability of the event 'not E' = ...

(ii) The probability of an event that cannot happen is .... Such an event is called ....

(iii) The probability of an event that is certain to happen is .... Such an event is called ....

(iv) The sum of the probabilities of all the elementary events of an experiment is ....

(v) The probability of an event is greater than or equal to .... and less than or equal to ....

Sol. : **(i)** 1, Because

$P(E) + P(\text{not } E) = 1$

(ii) 0 Impossible event

(iii) 1, Sure event

(iv) 1

(v) 0, 1 [$\because$ Probability is always between 0 and 1]

2. **Which of the following experiments have equally likely outcomes? Explain.**

(i) A driver attempts to start a car. The car starts or does not start.

(ii) A player attempts to shoot a basketball. She/he shoots or misses the shot.

(iii) A trial is made to answer a true-false question. The answer is right or wrong.

(iv) A baby is born. It is a boy or a girl.

Sol. : **(i)** is not equally likely, because normally the car starts moving but due to some fault the car does not start moving.

(ii) is not equally likely, because being able to put the ball in the basket depends on many factors like the player being able to put the ball, the quality of the basket ball, etc.

(iii) is likely, since the answer is correct, then it cannot be wrong and if it is wrong, it cannot be correct. Hence, there is an equal chance of being either true or false.

(iv) is equally likely, because when a child is born, it has the same probability of being a boy or a girl.

3. **Why is tossing a coin considered to be a fair way of deciding which team should get the ball at the beginning of a football game?**

Sol. : When we toss a coin, we get either head or tail which is equally likely. Hence, the result of the coin is completely one-sided or free from bias.

4. **Which of the following cannot be the probability of an event ?**

(a) $\frac{2}{3}$ **(b)** – 1.5

(c) 15% **(d) 0.7**

Sol. : (b) – 1.5, the probability of an event cannot be negative in any case, and all other alternatives lie between $0 \leq P(E) \leq 1$.

5. **If $P(E)$ = 0.05, what is the probability of 'not E' ?**

Sol. : Given, $P(E) = 0.05$

We know that, $P(E) + P(\overline{E}) = 1$

$$0.05 + P(\overline{E}) = 1$$

$$\therefore \quad P(\overline{E}) = 1 - 0.05 = 0.95$$

6. A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out :
(i) an orange flavoured candy?
(ii) a lemon flavoured candy ?

Sol. : A bag contains only lemon-flavoured candies. So the bag is not orange-flavoured candies.

(i) Here number of orange flavoured candies = 0

$\therefore$ P(orange flavoured candy) = 0

(ii) Here the bag contains only lemon flavoured candies.

$\therefore$ P (lemon flavoured candies) = 1

7. It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?

Sol. : Let, E = Event of two students not having on the same day

$$\therefore \quad P(E) = 0.992$$

$$\because \quad P(E) + P(\overline{E}) = 1$$

$$0.992 + P(\overline{E}) = 1$$

$$P(\overline{E}) = 1 - 0.992 = 0.008$$

Hence, the probability that the 2 students have the same birthday

= 0.008

8. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is (i) red ? (ii) not red?

Sol. : Total numbers of balls in bag $n(S) = 3 + 5 = 8$

Here,Number of red balls = 3

Number of black balls = 5

Let, E = event of drawing a red ball

$$\therefore \quad n(E) = 3$$

(i) Probability of drawing red ball

$$\text{P}(E) = \frac{n(E)}{n(S)} = \frac{3}{8}$$

(ii) Probability of not drawing red ball

$$= 1 - P(E)$$

$$= 1 - \frac{3}{8} = \frac{8-3}{8} = \frac{5}{8}$$

9. A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be : (i) red? (ii) white? (iii) not green?

Sol. : Total number of marbles in the box

$$n(S) = 5 + 8 + 4$$

$$n(S) = 17$$

Here,Number of red marbles = 5

Number of white marbles = 8

Number of green marbles = 4

(i) Probability of drawing red marbles = $\frac{5}{17}$

(ii) Probability of drawing white marbles = $\frac{8}{17}$

(iii) Probability of not drawing green marbles = Probability of drawing black or white marbles

$$\frac{5}{17} + \frac{8}{17} = \frac{13}{17}.$$

10. A piggy bank contains hundred 50p coins, fifty ₹ 1 coins, twenty ₹ 2 coins and ten ₹ 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin (i) will be a 50p coin? (ii) will not be a ₹ 5 coin?

Sol. : Given, number of 50 paise coins = 100

Number of ₹ 1 coins = 50

Number of ₹ 2 coins = 20

Number of ₹ 5 coins = 10

$\therefore$ Total coins $n(S) = 100 + 50 + 20 - 10$

$= 180$

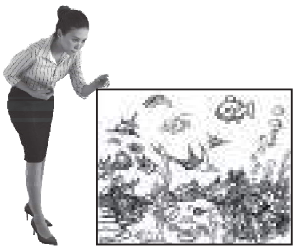
(i) $P(\text{Coin of 50p}) = \frac{100}{180} = \frac{5}{9}$

(ii) $\because P(\text{Coin of ₹ 5}) = \frac{10}{180} = \frac{1}{18}$

$\therefore P(\text{Not coin of ₹ 5}) = 1 - P(\text{Coin of ₹ 5})$

$$= 1 - \frac{1}{18} = \frac{18-1}{18} = \frac{17}{18}$$

11. Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 3 male fish and 8 female fish. What is the probability that the fish taken out is a male fish?



Sol. : Number of total fish in aquarium

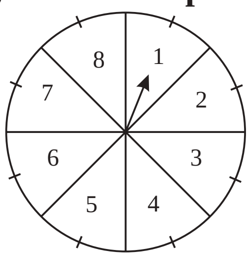
$= 5 + 8 = 13$

Number of male fish = 5

Number of female fish = 8

$\therefore \quad P(\text{Male fish}) = \frac{5}{13}$

12. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7 and 8 and these are equally likely outcomes. What is the probability that it will point at :



(i) 8 ?

(ii) an odd number ?

(iii) a number greater than 2 ?

(iv) a number less than 9 ?

Sol. : Number of total points in the circle $n(S) = 8$

(i) Let, E_1 = event of arrow coming on 8

then, $n(E_1) = 1$

$\therefore$ Probability of arrow coming on 8 $P(E) = \frac{n(E_1)}{n(S)} = \frac{1}{8}$

(ii) Let, E_2 = Event of arrow coming on one odd number

= 1, 3, 5, 7

$\therefore \quad n(E_2) = 4$

Probability of arrow coming on odd number

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

(iii) Let, E_3 = Event of arrow coming on a number greater than 2

= {3, 4, 5, 6, 7, 8}

$\therefore \quad n(E_3) = 6$

$\therefore$ Probability of arrow coming on a number greater than 2

$$= \frac{n(E_3)}{n(S)} = \frac{6}{8} = \frac{3}{4}$$

(iv) Let, E_4 = Event of arrow coming on a number less than 9

= {1, 2, 3, 4, 5, 6, 7, 8}

$\therefore \quad n(E_4) = 8$

∴ Probability of arrow coming on a number less than 9

$$P(E_4) = \frac{n(E_4)}{n(S)} = \frac{8}{8} = 1$$

13. A die is thrown once. Find the probability of getting :

(i) a prime number

(ii) a number lying between 2 and 6

(iii) an odd number.

Sol. : The possible outcomes of a single throw of a die are 1, 2, 3, 4, 5 and 6.

∴ $n(S) = 6$

(i) Let, E_1 = Event of getting a prime number

= (2, 3, 5)

∴ $n(E_1) = 3$

∵ $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

(ii) Let, E_2 = Event of a number lying between 2 and 6

= (3, 4, 5)

∴ $n(E_2) = 3$

∴ Probability of a number lying between 2 and 6

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

(iii) Let, E_3 = Event of getting an odd number

= (1, 3, 5)

∴ $n(E_3) = 3$

∴ Probability of getting an odd number

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

14. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting :

(i) a king of red colour

(ii) a face card

(iii) a red face card

(iv) the jack of hearts

(v) a spade

(vi) the queen of diamonds.

Sol. : Total number of cards in the deck $n(S) = 52$

(i) E_1 = Event of getting red king

∴ $n(E_1) = 2$

[∵ A deck of cards consists of 26 red and 26 black cards, in which there are 4 kings, and in which there are 2 red and 2 black kings.]

∴ Probability of getting red king

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

(ii) Let, E_2 = Event of a face card

$\therefore \quad n(E_2) = 12$

[$\because$ There are 4 kings, 4 queens and 4 slaves face cards in a deck of cards.]

$\therefore$ Probability of a face card

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{12}{52} = \frac{3}{13}$$

(iii) Let, E_3 = Event of getting a red face card

$\therefore \quad n(E_3) = 6$

$\therefore$ Probability of getting a red face card

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{6}{52}, \frac{3}{26}$$

(iv) Let, E_4 = Event of getting the jack of hearts

$\therefore \quad n(E_4) = 1$

$\therefore$ Probability of getting the jack of hearts

$$P(E_4) = \frac{n(E_4)}{n(S)} = \frac{1}{52}$$

(v) Let, E_5 = Event of getting a spade

$\therefore \quad n(E_5) = 13$

[$\because$ A deck of cards consists of 13 spades, 13 hearts, 13 clubs and 13 diamonds.]

$\therefore$ Probability of getting a spade

$$P(E_5) = \frac{n(E_5)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

(vi) Let, E_6 = Event of drawing the queen of diamond

$\therefore \quad n(E_6) = 1$

$\therefore$ Probability of drawing the queen of diamond

$$P(E_6) = \frac{n(E_6)}{n(S)} = \frac{1}{52}$$

15. **Five cards – the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random.**
(i) What is the probability that the card is the queen?
(ii) If the queen drawn and put aside, what is the probability that the second card picked up is (a) an ace? (b) a queen?

Sol. : **(i)** Total number of cards = 5

$\therefore$ P (Probability of drawing a queen) = $\frac{1}{5}$

(ii) Let, the queen be taken out and kept aside, Then, number of cards left = 4

(a) P (Probability of drawing an ace) = $\frac{1}{4}$

(b)P (Probability of drawing a queen) = $\frac{0}{4}$ = 0

16. **12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.**

Sol. : Given, Number of defective pen = 12

Number of good pen = 132

∴ Total pen [$n(S)$] = 12 + 132 = 144

Let, E = Event of good pen

Then, $n(E)$ = 132

∴ Probability of choosing a good pen

$$P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{132}{144} = \frac{11}{12}$$

17. **(i) A lot of 20 bulbs contain 4 defective ones, One bulb is drawn at random from the lot. What is the probability that this bulb is defective?**

(ii)Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?

Sol. : **(i)** Given, number of total bulbs $n(S)$ = 20

Let, E_1 = Event of choosing defective bulb

$n(E_1) = 4$

∴ Probability of choosing a defective bulb= $\frac{n(E_1)}{n(S)} = \frac{4}{20} = \frac{1}{5}$

(ii) Suppose a good bulb is taken out and kept outside, then there are 15 good bulbs and 4 bad bulbs left in the group.

Let, E_2 = Event of choosing a good bulb

$n(E_2) = 15$

∴ Probability of choosing a not defective bulb

$$= \frac{n(E_2)}{n(S_1)} = \frac{15}{19}$$

18. **A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that bears (i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5.**

Sol. : Total discs in the box = 90

∴ $n(S) = 90$

(i) Let, E_1 = Event of choosing a two digit number

= {10, 11, 12, ..., 90}

∴ $n(E_1) = 81$

Probability of choosing a two digit number

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{81}{90} = \frac{9}{10}$$

(ii) Let, E_2 = Event of choosing a perfect square number
= (1, 4, 9, 16, 25, 36, 49, 64, 81)

$\therefore$ $n(E_2) = 9$

$\therefore$ Probability of choosing a perfect square number

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{9}{90} = \frac{1}{10}$$

(iii) Let, E_3 = Event of choosing a number divisible by 5
= {5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90}

$\therefore$ $n(E_3) = 18$

$\therefore$ Probability of choosing a number divisible by 5

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{18}{90} = \frac{1}{5}$$

19. A child has a die whose six faces show the letters as given below :

A	B	C	D	E	A

The die is thrown once. What is the probability of getting (i) A? (ii) D ?

Sol. : Total number of outcomes of a 6 faced dice $n(S) = 6$

(i) Let, E_1 = Event of getting letter A

$n(E_1) = 2$

$\therefore$ Probability of getting letter A

$$= \frac{n(E_1)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

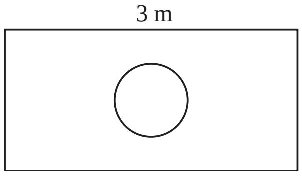
(ii) Let, E_2 = Event of getting letter D

$\therefore$ $n(E_2) = 1$

$\therefore$ Probability of getting letter D

$$= \frac{n(E_2)}{n(s)} = \frac{1}{6}$$

20. Suppose you drop a die at random on the rectangular region shown in figure below. What is the probability that it will land inside the circle with diameter 1m?



Sol. : Total number of outcomes

$n(S)$ = Area of rectangle
$= 3 \times 2 = 6\ m^2$

And the number of favourable outcomes

Area of circle $n(E) = \pi r^2 = \pi \left(\frac{1}{2}\right)^2 = \frac{\pi}{4} m^2$

$\therefore$ Required probability, $P(E) = \frac{n(E)}{n(S)}\ \frac{\pi'/4}{6} = \frac{\pi}{24}$

21. A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that :

(i) She will buy it ?

(ii)She will not buy it ?

Sol. : Total number of pens = 144

Number of defective pen = 20

Let, E_1 = Event of choosing a good pen

$\therefore$ $n(E_1) = 144 - 20 = 124$

(i) If the pen is good he will buy the pen

$\therefore$Required probability $P(E_1)$

$$= \frac{n(E_1)}{n(S)} = \frac{124}{144} = \frac{31}{36}$$

(ii) If the pen is not good, he will not buy the pen

$\therefore$Required probability $= 1 - P(E)$

$$= 1 - \frac{31}{36} = \frac{36-31}{36} = \frac{5}{36}$$

22. Two dice one blue and one grey, are thrown at the same time.

(i) Complete the following tables :

Event 'Sum of 2 dice'	Probability
2	$\frac{1}{36}$
3	
4	
5	
6	
7	
8	$\frac{5}{36}$
9	
10	
11	
12	$\frac{1}{36}$

(ii) A student argues that there are 11 possible outcomes 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. Therefore, each of them has a probability $\frac{1}{11}$. Do you agree with this argument? Justify your answer.

Sol. : (i) Following are the total possible outcomes of throwing two dice :

{(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6) (2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6) (3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6) (4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6) (5, 1) (5, 2) (5, 3) (5, 4) (5, 5)

(5, 6) (6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)}

∴ $n(S) = 36$

(a) Let, E_1 = Sum of two dice is 3

= {(1, 2) (2, 1)}

$n(E_1) = 2$

∴ $P(E_1) = \frac{2}{36} = \frac{1}{18}$

(b) Let, E_2 = Sum of two dice is 4 = {(1, 3) (2, 2) (3, 1)}

∴ $n(E_2) = 3$

∴ $P(E_2) = \frac{3}{36} = \frac{1}{12}$

(c) Let, E_3 = Sum of two dice is 5 = {(1, 4)(2, 3)(3, 2)(4, 1)}

$n(E_3) = 4$

∴ $P(E_3) = \frac{4}{36} = \frac{1}{9}$

(d) Let, E_4 = Sum of two dice is 6

= {(1, 5)(2, 4)(3, 3)(4, 2)(5, 1)}

∴ $n(E_4) = 5$

∴ $P(E_4) = \frac{5}{36}$

(e) Let, E_5 = Sum of two dice is 7 = {(1, 6)(2, 5)(3, 4)(4, 3) (5, 2) (6, 1)}

$n(E_5) = 6$

∴ $P(E_5) = \frac{6}{36} = \frac{1}{6}$

(f) Let, E_6 = Sum of two dice is 8 ={(2, 6)(3, 5)(4, 4)(5, 3)(6, 2)}

$n(E_6) = 5$

∴ $P(E_6) = \frac{5}{36}$

(g) Let, E_7 = Sum of two dice is 9 = {(3, 6)(4, 5)(5, 4)(6, 3)}

$n(E_7) = 4$

∴ $P(E_7) = \frac{4}{36} = \frac{1}{9}$

(h) Let, E_8 = Sum of two dice is 10 = {(4 6) (5, 5) (6, 4)}

$n(E_8) = 3$

∴ $P(E_8) = \frac{3}{36} = \frac{1}{12}$

(i) Let, E_9 = Sum of two dice is 11 = {(5, 6) (6, 5)}

$n(E_9) = 2$

∴ $P(E_9) = \frac{2}{36} = \frac{1}{18}$

(j) Let, E_{10} = Sum of two dice is 12 = {(6, 6)}

∴ $n(E_{10}) = 1$

∴ $P(E_{10}) = \frac{1}{36}$

(ii) No, we do not agree with this fact because these events are not equally likely.

23. A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result *i.e.*, three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

Sol. : Total possible outcomes of tossing a coin three times

= {(HHH), (HHT), (HTH), (THH), (THT), (TTH), (HTT), (TTT)}

$\therefore \quad n(S) = 8$

If all the tosses do not have the same result, Hanif will lose the game.

or, {(HTT) (HHT) (THT) (HTH) (TTH) (THH)}

$\therefore \quad n(E) = 6$

$\therefore$ Required probability $P(E) = \frac{n(E)}{n(S)} = \frac{6}{8} = \frac{3}{4}$

24. A die is thrown twice. What is the probability that :

(i) 5 will not come up either time?

(ii) 5 will come up at least once?

Sol. : Total possible outcomes $n(S) = 36$

Let, E = Event of getting 5

= {(1, 5)(2, 5)(3, 5)(4, 5)(5, 5)(6, 5) (5, 1) (5, 2) (5, 3)(5, 4)(5, 6)}

$\therefore \quad n(E) = 11$

and $\quad \overline{E}$ = Event of not getting 5

$\therefore \quad n(\overline{E}) = n(S) - n(E) = 36 - 11 = 25$

(i) $\therefore$ Probability of 5 will not come up either time

$$P(E) = \frac{n(\overline{E})}{n(S)} = \frac{25}{36}$$

(ii) Probability of 5 will come up at least once

$$P(E) = \frac{n(E)}{n(S)} = \frac{11}{36}$$

25. Which of the following arguments are correct and which are not correct? Give reasons for your answer.

(i) If two coins are tossed simultaneously there are three possible outcomes–two heads, two tails or one of each. Therefore, for each of these outcomes, the probability is $\frac{1}{3}$.

(ii)If a die is thrown, there are two possible outcomes–an odd number or an even number. Therefore, the probability of getting an odd number is $\frac{1}{2}$.

Sol. : **(i)** False, possible outcomes of tossing two coins ={(H,H), (H, T), (T, H), (T, T)}

then, $\quad P(\text{H, H}) = \frac{1}{4}, P(\text{T, T}) = \frac{1}{4}$

and, $P\{(\text{H, T}), (\text{T, H})\} = \frac{2}{4} = \frac{1}{2}$

(ii) True, total possible outcomes of dice

= {1, 2, 3, 4, 5, 6}

Odd numbers = {1, 3, 5}

Even numbers = {2, 4, 6}

∴ Probability of getting an odd number

$$= \frac{3}{6} = \frac{1}{2}$$

Probability of getting an even number

$$= \frac{3}{6} = \frac{1}{2}$$

Hence, both have equal probability $\frac{1}{2}$.